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The Gender Gap in University Participation: What role do Skills and Parents Play?

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Abstract

University participation among women has been increasing over the last 3 decades such that now in Canada more than half of all new degrees are awarded to women. Recent research has suggested that boys are also falling behind in their grades and educational aspirations during high school. Both grades and aspirations reflect many different individual characteristics and socio-economic circumstances. To uncover the deeper determinants of the gender gap in university participation, I use the Youth in Transition Survey to estimate a factor model based on a framework developed by Foley, Gallipoli, and Green (2014). I use that model to identify and quantify the impact of three factors: cognitive skills, non-cognitive skills and parental valuations of education (PVE). I find that all three factors play an important role in explaining both the level and the gap in university participation. The factor structure as a whole accounts for 88 percent of the gender gap, and of that the PVE factor accounts for 28 percent. This result suggests that parents play a larger role than what is implied by decompositions employing only observed determinants.

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1 Introduction

While the fraction of Canadians with university degrees was growing steadily during the second half of the 20th century, growth in female graduates far out paced that of men (Christofides et al., 2010). In 2013, 60% of graduates with bachelors degrees in Canada were women (OECD, 2016). Indeed, women have overtaken men in university enrollment in most OECD countries.

Using data from the United States, economists have proposed several explanations for the gender gap in university participation, and academic achievement more generally. These include gender differences in non-cognitive skills (Jacob, 2002; Becker et al., 2010; Conger and Long, 2010), job opportunities and the returns to schooling, (Goldin, 1995), as well as, aspirations and plans for the future (Fortin et al., 2015).

Although, the gender gap in university graduation is larger in Canada (OECD, 2016) than in the U.S., arguably less has been written about this topic using Canadian data. Frenette and Zeman (2007), who investigate the gender gap in university participation using the Youth in Transition Survey (YITS), are a notable exception. The YITS is a longitudinal survey that follows a cohort of Canadian youth beginning when they were aged 15, and which also includes a parental survey. Frenette and Zeman (2007) perform a Oaxaca -Blinder decomposition to estimate how much of the gap can be attributed to various observed characteristics. Grades, scores on a reading test, study habits, and parental aspirations for their children's education are among the most important contributing variables.

Grades and parental aspirations, however, are variables that reflect many different contributing factors, some of which are not directly observed in data. The goal in this paper is to identify and quantify the underlying factors that explain the gender gap in university participation. Like Frenette and Zeman (2007), I use the YITS. However, I use one more cycle of the survey measuring participation at age 21 rather than 19.[1] In that data, 53% of girls and 48% of boys attended

[1]Because the data includes youth from Ontario for whom a fifth year of high school was possible, participation rates increase considerably between age 19 and 21.

university, leading to a gap of 15 percentage points.

To identify the more fundamental determinants of this participation gap, I use a factor model developed by Foley, Gallipoli, and Green (2014)–hereafter FGG. The model in FGG builds on the empirical approach introduced by Carneiro et al. (2003) and Cunha et al. (2005), and which explores the socioeconomic gradient in dropping out of high school. The key idea is that individuals know and understand some of the factors that affect educational outcomes, but which are unobserved in data. The wide variety of measures of ability, behaviour, and attitudes that are included in rich data sets, such as the YITS, can be used to extract those factors. Following the literature emphasizing the importance of skills (Cunha and Heckman, 2007; Cunha et al., 2010), FGG consider cognitive and non-cognitive skills as important factors determining the decision to drop out. One of the contributions of FGG is to introduce a third factor, called the parental valuation of education (PVE), that reflects differences in how parents value education, either in a pecuniary or non-pecuniary way.

The PVE factor is by construction correlated with parents' aspirations measured by responses to a question that asked parents to indicate how much schooling they hope their child obtains. When parents respond to such questions, their answers will reflect not only their own valuation but also what they know about their child's ability and motivations. Similarly, variables such as high school grades reflect the students' skill and effort, but can also be influenced by parents in ways that depend on how the parents value education. To separately identify the impact of the three factors–cognitive and non-cognitive skill, and parental valuations–the factor model in FGG, and in this paper, employs covariance restrictions on noisy measures of those factors. Many of the measurement variables in FGG explain large portions of the gap in Frenette and Zeman (2007). As such, this factor model is well suited to help unpack the determinants of the gender gap in university participation.

I estimate the model allowing the distribution of the factors and their impact on outcomes to vary across gender. For both boys and girls, I find that all three factors have a significant impact on university participation. Cognitive skills have the largest impact, increasing the probability of attendance by more than .50 when comparing the lowest to the highest skill level. The impact of parental valuations varies across the two skill levels. It can be almost as large as the cognitive skill gradient. For example, conditional on a medium level of both cognitive and non-cognitive skills, a high level of the PVE factor increases the probability of attendance by nearly 40 percentage points, for both boys and girls.

I also find that once I have controlled for the factors, the direct effect of gender, represented by an intercept shift, is no longer statistically significant. The predicted probability of attending university among those with the highest cognitive skill level is nearly identical for both boys and girls. Gender gaps do persist at some of the lower skill levels, particularly when the parental valuation factor is high, however, the model predicts that a relatively small fraction of the data fall into those categories.

Overall, the results suggest that girls attend university more often than boys because they have higher levels of all three factors. To quantify the importance of each factor, I perform a decomposition. As a whole the factor structure can account for 88 per cent of the gap. While each factor plays an important role, the cognitive factor explains the largest portion, amounting to roughly 40 per cent.

A key finding is that the parental valuation factor accounts for 28 per cent of the gender gap in participation. This is a much larger impact than one would find by restricting attention to observed variables such as parental aspirations. Parents influence their children's behavior through several channels. The factor model makes it possible to quantify, and aggregate into the PVE factor, the different channels through which parents' valuations operate.

The rest of the paper proceeds by first describing previous research on gender gaps in schooling and academic achievement. I, then, describe the Youth in Transition Survey and the data I use from that survey. The next section describes the factor model, and the estimation method. The results section commences with some reduced form regressions that describe key relationships in the data. The results from the factor model are then presented, followed by the decomposition of the university participation gap. The final part of the results section includes a discussion of why the parental valuation factor is higher among girls. Here, I address the issue of potential bias from unobserved ability. Finally, before concluding, I investigate whether the reasons parents give for their aspirations provide any clues as to why the PVE factor is, on average, higher for girls.

1.1 Related Research

Much of the economic literature investigating gender gaps in schooling and academic achievement has focused on explaining the observed changes over time. Factors related to the improving labour market conditions for women figure prominently in explanations of the trend. In cross-sectional data, differences in skills and behaviour emerge as more important contributors to gender gaps within cohorts. There is also a growing literature investigating the role that parents and in particular family disadvantage plays in explaining why females are more likely to attend university than males.

To investigate the evolving pattern of gender gaps in college attendance and graduation over the 20th century, Goldin et al. (2006) combine analyses from several U.S. data sets. They argue that once the restrictions posed by gender-norms were loosened, higher wage premiums encouraged women to attend college, and to enter career oriented programs, at increasing rates. They further suggest that women eventually became a majority, in part, because young men face higher costs due to their noncognitive and behavioural disadvantages.

Goldin et al. (2006) also point out that a 'proximate determinant' of the college gender gap is higher academic achievement among girls. Fortin et al. (2015) directly investigate the widening gender gap in academic achievement among high school students. Using data from 'Monitoring the Future' (MTF) surveys, they show that girls have become increasingly more likely to achieve A's. They emphasize the importance of 'plans for the future' in explaining the widening gender gap at the top of the achievement distribution. In particular, girls are more likely than boys to aim for graduate and professional degrees.

Data constraints make it relatively difficult to study similar trends in Canada. Christofides et al. (2010) combine data from the Survey of Labour and Income Dynamics, and the Survey of Consumer Finances to investigate changes in the relative university participation rates among boys and girls from 1977 to 2005. They conclude that differences in the university wage-premium explain most of the changes in the participation gap during this period. A key limitation to these data sets is that they contain no information about youths' skills or academic achievement.

Although no nation-wide Canadian data set exists that combines family background, skills and schooling outcomes for more than one cohort, Card et al. (2011) make use of administrative data from the Ontario applications system for the high school graduating cohorts of 1991 to 2004. During this relatively short time horizon, several important policy changes occurred in Ontario, including the deregulation of tuition fees and the shortening of the high school diploma from five to four years. While the gender gap in application rates increased by 4 percentage points, from .9 to .13, some of this was driven by differential changes in the size of the cohorts. Although this data does not contain detailed information about family background, Card et al. (2011) perform a school level analysis linking the surrounding neighbourhood characteristics to the school. This reveals that school characteristics explain little, but a gender gap, favouring girls, appears as early as grade 9 in selecting academic track math courses.

In studies that use data from a single cohort, differences in skills and behaviour play a larger role in explaining gender gaps. Decomposing the gender gap in the National Education Longitudinal Survey (NELS1988) cohort, Jacob (2002) attributes the largest fraction of the gap to non-cognitive skills measured by grades and behavioural problems. However, Jacob (2002) also finds that university premiums are the second largest contributing factor in this cohort of American youth.

Similarly, in Canadian data, when controls for skills and behaviour are available, these factors are more important than wage premiums in predicting the gender gap in university participation. Using the Youth in Transition Survey, Frenette and Zeman (2007) perform a Oaxaca-Blinder decomposition, generally, finding that grades and performance on a standardized reading skills test are the most important explanatory variables. Study habits and parental aspirations are also significant contributors. Wage premiums are less important, explaining roughly 5 per cent of the gap.

Like Frenette and Zeman (2007), I also use the YITS, however, instead of using the third cycle, I use the fourth, which was not available when Frenette and Zeman (2007) were writing. In the fourth cycle, the youth are aged 21. In general, university participation continues to increase between ages 19 and 21. More importantly, my work extends Frenette and Zeman (2007) by investigating the deeper determinants of the gender gap, identifying the unobserved factors that drive the observed relationships. My empirical approach is explicit about how variables such as grades and parental expectations are related to each other, the participation decision, and the underlying unobserved factors. The results in this paper confirm the importance of skills, but also point toward a larger role for parents than is implied by Frenette and Zeman (2007).

Christofides et al. (2008a,b) also use the YITS, attempting to relate the gender gap in youths' aspirations to the gender gap in university participation. Their approach does not allow for any dependence between the unobserved components of youths' reported aspirations and university attendance. I explicitly model that relationship in this paper, which provides evidence about the channels through which aspirations may impact participation.

Much of the research investigating whether boys and girls are differentially impacted by their parents and family environment focuses on the early years (For example, Bertrand and Pan, 2013; Baker and Milligan, 2013). However, increasingly, attention is directed toward the role that parents' play in explaining gender gaps in secondary and post-secondary schooling outcomes. Buchmann and DiPrete (2006) investigate whether increasing levels of parental education can explain why women have overtaken men in educational attainment in the U.S., either because educated parents value equality between genders more highly or because educated mothers have stronger role model effects on daughters. The find little support for these hypotheses and instead note that boys growing up in less educated households are falling behind.

Autor et al. (2016) also investigate whether boys are affected more by growing up in economically disadvantaged environments. Making use of linked administrative data from Florida, they find that the gender gaps, which emerge in Kindergarten are wider among those from lower socioeconomic backgrounds. That socioeconomic gradient, in the gender gap, is observed persistently throughout secondary schooling and high school graduation.

Although their data does not measure outcomes beyond high school, Autor et al. (2016) posit that the patterns they document would extend into adulthood. In contrast, in Danish administrative data, Brenøe and Lundberg (2016) find evidence suggesting the opposite. Like in the U.S. data, gender gaps in adolescent outcomes, such as completing grade nine on time, are also exacerbated in less educated households in the Danish data. However, Brenøe and Lundberg (2016) go on to show that when considering educational attainment in adulthood, the pattern substantially changes, and also differs across mothers' and fathers' education levels. By age 27, the gender gap, which favours women, in years of schooling completed is wider for those whose mother has a university degree. In contrast, that gap narrows among those whose father has a university degree. Brenøe and Lundberg (2016) suggest that role model effects may in part explain why women may benefit relatively more from maternal education.

This paper is among the first to contribute Canadian evidence to this literature. Specifically, the aim is to identify a particular channel through which parents affect university participation, beyond the impact they have on skills development. My results confirm that boys do lag behind girls in their skill levels. I further demonstrate that after controlling for those skill differences, parents still have a differential impact on boys and girls.

2 Data

The Youth in Transition Survey (YITS), a longitudinal survey of youth, is among the only data sets in Canada that combines information about academic achievement, attitudes and motivations, secondary and post-secondary schooling outcomes, and family background. Cohort A, which I use in this paper, is a nationally representative sample of Canadian youth born in 1984. The original sample, consisting of 29,687 students, was selected in two-stages. In the first stage, high schools were randomly selected from a list generated by the provinces. In the second stage, students were selected from within the schools to facilitate school-level analysis.[2] Because some provinces and linguistic groups were over-sampled, the within-school sampling rate ranged from less than 10 percent to a census of the 15 year-olds. In all of the results I report, I use weights provided by Statistics Canada that account for over-sampling, as well as attrition.

In 2000, during the first cycle of the survey, students completed the Program for International Student Assessment (PISA) reading test. PISA tests, which are coordinated by the OECD, are designed to produce internationally comparable measures of knowledge and skills. A random subset, amounting to slightly more than half, of the students also wrote the math or science PISA tests. Because the sample sizes are so much smaller, I do not use those scores in the main estimation but make use of them in a robustness analysis.

The YITS also includes a parents' survey completed by the parent or guardian who identified him or herself as 'most knowledgeable' about the child. Parents provided information about themselves and their spouses, including their education and income. Parents also answered questions about their attitudes and behaviour as related to their children's education. The final component of the first cycle of YITS data collection is a school administrators survey, which collected information describing the schools' characteristics and resources.

Only the students were followed in the longitudinal component, and they were interviewed

[2]Schools were excluded from the sample if fewer than 3 students were present or likely to respond to the survey. Schools for children with severe learning disabilities, schools for blind and deaf students and schools on First Nations reserves were also excluded.

every two years. I combine data from the first cycle defining individual and family characteristics, with data from the fourth cycle, collected in 2006 when the students were 21 years of age. University participation is then defined as ever having enrolled in a program that leads to a Bachelors degree by age 21.

The analysis sample used in this paper is restricted to youth who completed the fourth survey, and whose parents completed the survey in cycle one. The final sample size among those with non-missing data is 5303 girls and 4507 boys. That there are more girls than boys in the sample is indicative of differential attrition. Although the weights do account for attrition based on observed characteristics, it is not possible to rule out the possibility of non-random attrition based on unobserved characteristics. Motte et al. (2008) provide additional information about the YITS and attrition.

For the pooled sample, and for boys and girls separately, in Table 1, I report the means for the participation outcomes, as well as all the variables that are used as measurements in the factor model. The definition of the variables and the role they play in the model are described in the next section that outlines my empirical approach. The means for the variables which characterize the youths' socioeconomic and family background are reported in Table 2, and their definition is described in the opening of the results section.

3 Empirical Approach

To identify the unobserved factors that determine university participation, and in turn, the gender gap, I employ a factor model, which FGG use to explain dropping out of high school. The model is an extension of the approach developed by Carneiro et al. (2003) and Cunha et al. (2005), hereafter CHH and CHN. In this section, I describe how I apply this empirical approach to the problem of explaining the gender gap in university participation.

To begin, it is useful to separate the factors that determine whether an individual, of gender g, is a university participant by age 21 into those that are observed in data, Z_i, and a second set which are not ε^g_{i0}. In particular, Z_i represents the set of background characteristics measured when the youth is age-15.

Assuming that the underlying utility function is approximately linear, then the university participation decision can be represented by a latent variable model:

$$I^g_i = \gamma^g_0 + \gamma^g_z Z_i + \varepsilon^g_{i0}$$

If $I^g_i \geq 0$ then the individual is observed as an university participant in the data.

Since Z_i is data measured at age-15, ε^g_{i0} contains not only unobserved characteristics, but also any new information or changes in characteristics that occur between age-15 and the participation outcome. As CHH and CHN point out, factors that are unobserved in data are not necessarily unknown to the age-15 youth. If Θ^g_i is the vector of unobserved characteristics which are known to the age-15 youth, and Λ^g_0 are the returns to those characteristics, then the participation index can be re-written as:

$$\begin{aligned} I^g_i &= \gamma^g_0 + \gamma_z Z_i + \Lambda^g_0 \Theta^g_i + u^g_{i0} \\ &= \gamma^g_0 + \gamma_z Z_i + \lambda^g_{0\theta_1}\theta_{i1} + \lambda^g_{0\theta_2}\theta_{i2} + \lambda^g_{0v_p} v_{ip} + u^g_{i0} \end{aligned} \tag{1}$$

The second equality follows because Θ^g_i is a vector of three latent factors: $\{\theta_{i1}, \theta_{i2}, v_{ip}\}$. As in FGG, these factors are labelled as ‘cognitive skills’, ‘non-cognitive skills’, and ‘parental valuation of education’, respectively. The three factors are assumed to be mutually independent.

In this specification, the error term u^g_{i0} represents all the factors that affect university participation, which the youth at age 15 can not foresee, or does not understand, and which are not observed in data. As such, when those age-15 youth are asked, ‘What is the highest level of education you would like to get?’, their response should reflect Z_i and Θ^g_i but not u^g_{i0}. Thus, an index function for youths’ aspirations is:

$$yasp^g_i = \beta^g_{10} + \beta_{1z} Z_i + \lambda^g_{1\theta_1}\theta_{i1} + \lambda^g_{1\theta_2}\theta_{i2} + \lambda^g_{1v_p} v_{ip} + u^g_{i1} \tag{2}$$

Although, the factors are not directly observed, the data contains variables which act as noisy measures of the factors. Interpretation of the factors depends on the measurement system, and the restrictions that are used to identify the factors.

The first factor, θ_{i1}, is measured using quartiles of the PISA reading score. The index for those PISA quartiles is[3]:

$$PISA_i^g \quad = \quad \beta_{20}^g + \beta_{2w} W_i + \lambda_{2\theta_1}^g \theta_{i1} + u_{i2}^g \tag{3}$$

This specification, which implies that the factor loads on both θ_{i2}, and υ_{ip} are zero in the PISA measurement equation, is an important restriction and affects the way the factors are interpreted.

In FGG, we describe a model of how ability evolves during childhood giving rise to the interpretation of θ_{i1} as the stock of ability at age-15. It can be thought of as a sufficient statistic, capturing all of the inputs and investments that generate a youths' level of ability when she is age 15. This interpretation is invalid if, for example, the path by which these skills were obtained matters above and beyond the skills themselves.

Although parental inputs and attitudes almost certainly influence the development of θ_{i1}, conditional on that ability, as we argue in FGG, parents are unlikely to influence the PISA test score directly because it is a one-time, low-stakes test. The PISA test is not used in assessing individual student performance, nor does it measure mastery of course curriculum. For these reasons, υ_{ip} does not enter the PISA equation.

There is evidence, however, that performance on low stakes tests does depend on personality characteristics that differ from the notion of intelligence, which might be typically thought of as 'cognitive skills' (Borghans et al., 2011). Low test scores may reflect a lack of motivation rather than skill. With this in mind, the cognitive factor, can not be interpreted as a cleanly identified

[3] I use the average across the 5 plausible values for the Reading test. Then, I generate quartiles across the full YITS sample, including non-responders to the fourth cycle and the parental survey. In other words, the quartiles are defined before any sample restrictions are made.

measure of intelligence or academic skill.

However, because the system of measurements is specified such that each factor is orthogonal to the others, the second factor θ_{i2}, which is labelled as 'non-cognitive' skills, will reflect factors that are not already captured by θ_{i1}. Since θ_{i2} reflects incremental skills, rather than the total level of skill, it can be thought of as a lower bound.

Defining and measuring non-cognitive skills is complicated by its multi-dimensional nature. While several taxonomies for personality characteristics exist, Almlund et al. (2011) suggest that the 'Big Five' factors are most widely accepted. This taxonomy is based on factor models that extract the 'common variance' among many different underlying facets. The five factors, with which the more specific facets are correlated, have been described by the traits: Openness to experience, Conscientiousness, Extraversion, Agreeableness, and Neuroticism.

As in FGG, the measures of θ_{i2} are selected to best reflect the concept of 'Conscientiousness'. This personalty trait is characterized by the adjectives: efficient, organized, planful, reliable, responsible, and thorough (McCrae and John, 1992). Measures of conscientiousness have also been found to predict educational outcomes, including grades in post-secondary schooling and years of education (Borghans et al., 2008; Almlund et al., 2011).

Although the YITS does not contain a specific scale for conscientiousness, following FGG, I use measures of self-reported behaviours that are related to the characteristic adjectives. The first measure is a variable that takes on the value one if the youth responded 'always' when asked how often the following statement applies: 'I complete my homework on time'. The underlying index function is:

$$hmwrk_i^g = \beta_{30}^g + \beta_{3w} W_i + \lambda_{3\theta_2}^g \theta_{i2} + \lambda_{3v_p}^g v_{ip} + u_{i3}^g \tag{4}$$

In FGG, and Heckman et al. (2006), the non-cognitive measures are not a function of the cognitive factor. I follow suit here. However, even after controlling for non-cognitive skills, parents' can influence the timely completion of homework by offering incentives or punishments and as such the parental valuation factor is included in the *hmwrk* equation.

The key measurement for the parental valuation of education is the parental aspirations question. The responding parent was asked 'What is the highest level of education that you hope your child will get?'. The corresponding variable is coded as equalling one if the parent responded either 'One university degree' or 'More than one university degree'. Because parents almost certainly take into account not only their own valuation of education but also their children's skills, the parental aspiration measurement is a function of all three factors:

$$parasp_i^g = \beta_{40}^g + \beta_{4z}Z_i + \lambda_{4\theta_1}^g\theta_{i1} + \lambda_{4\theta_2}^g\theta_{i2} + \lambda_{4v_p}^g v_{ip} + u_{i4}^g \tag{5}$$

Identification of the factors requires at least two measurements for each factor. For another measure of cognitive ability, I use overall high school grades reported by the youth at age 15. Grades generally reflect, not just academic skill, but also effort and behaviour in school. Moreover, since parents can become involved in their child's school work either directly or though encouragement, grades vary with all three factors[4]:

$$grades_i^g = \beta_{50}^g + \beta_{5w}W_i + \lambda_{5\theta_1}^g\theta_{i1} + \lambda_{5\theta_2}^g\theta_{i2} + \lambda_{5v_p}^g v_{ip} + u_{i5}^g \tag{6}$$

The second measure of non-cognitive skills is related to the 'thoroughness' aspect of Conscientiousness. This variable takes on the value one if the youth responded 'never' when asked how often the following statement was true, "I do as little work as possible; I just want to get by." The underlying index function is:

$$getby_i^g = \beta_6^g + \beta_{6w}W_i + \lambda_{6\theta_2}^g\theta_{i2} + \lambda_{6v_p}^g v_{ip} + u_{i6}^g \tag{7}$$

[4]In FGG, to help justify the inclusion of the parental valuation factor in the *grades* equation while it is excluded in the PISA reading score equation, we show that Math and Sciences grades are significantly related to both reading scores and parental aspirations. In contrast, PISA math and science scores, after controlling for the reading scores, are not related to parental aspirations.

Finally, the parental valuations of education are also measured by a variable that indicates whether parents have saved for their children's education. Specifically, parents are first asked "Have you (or your partner) done anything specific to ensure that your child will have money for further education after high school?" Having 'saved' means the parent further indicated that he or she had 'started a savings account', 'started a Registered Education Savings Plan (RESP)', 'set up a trust fund for this child', or 'made investments, such as mutual funds or Canada Savings Bonds'. Like the parental aspirations variable, the 'saved' index is a function of all three factors:

$$saved_i^g \;=\; \beta_7^g + \beta_{7z} Z_i + \lambda_{7\theta_1}^g \theta_{i1} + \lambda_{7\theta_2}^g \theta_{i2} + \lambda_{7v_p}^g v_{ip} + u_{i7}^g \tag{8}$$

The conditions under which the factors, their variances, and the loadings are identified are described in detailed in Carneiro et al. (2003). In particular, the factors are each mutually independent, with a mean of zero. The measurement errors (u^g) are also independent of the covariates, the factors and other errors. Under these conditions, the covariances among the measurements provide the identifying information. At a minimum, the number of measurements should be twice the number of factors, plus one. In this model there are three factors, and eight equations including the outcome equation.

Identification also requires further normalization. Since the factors have no natural scale, for each factor, one of the loadings is normalized to one. Here, the cognitive factor loading in the $PISA$ measurement is normalized, as are the non-cognitive and PVE loadings in the *hmwrk* and *parasp* equations, respectively. Finally, the restriction that there is one measurement which is dedicated to a single factor is necessary for identification.[5] That dedicated measure is the PISA equation which is a function of only the cognitive factor.

I estimate two versions of the model. The first is a 'constrained' model in which the parameters are the same for men and women, with the exception of the female intercept in each equation. The 'flexible' or 'unconstrained' model, allows the factor structure to differ across gen-

[5] This normalization is described in footnote 18 of Carneiro et al. (2003).

ders. Similar to the approach taken in Heckman and Singer (1984), the factors are specified as discrete variables, where one point of support is normalized to zero. Since these factors have no meaningful scale, the factor locations are the same for both sexes in the flexible model, however, the probability associated with each level depends on gender. Additionally, the factor loadings, or the impact the factors have on university participation, differ by gender.[6]

Both versions of the model are estimated by maximum likelihood. The likelihood function is defined conditional on each level of the factors, then weighted by the probability associated with each factor location and summed. An example contribution to the likelihood function, in the flexible model, is:

$$\begin{aligned}
\textstyle\sum_{\theta_1}\sum_{\theta_2}\sum_{v_p} p^g(\theta_1)\, p^g(\theta_2)\, p^g(v_p) \quad & F\left(\gamma_0^g + \gamma_z Z_i + \lambda_{0\theta_1}^g \theta_{i1} + \lambda_{0\theta_2}^g \theta_{i2} + \lambda_{0v_p}^g v_{ip}\right) * \\
& F\left(\beta_{10}^g + \beta_{1z} Z_i + \lambda_{1\theta_1}^g \theta_{i1} + \lambda_{1\theta_2}^g \theta_{i2} + \lambda_{1v_p}^g v_{ip}\right) * \\
& F\left(\sigma_p^{-1}\left[PISA_1 - \beta_{20}^g - \beta_{2w} W_i - \theta_{i1}\right]\right) * \\
& F\left(\beta_{30}^g + \beta_{3w} W_i + \theta_{i2} + \lambda_{3v_p}^g v_{ip}\right) * \\
& F\left(\beta_{40}^g + \beta_{4z} Z_i + \lambda_{4\theta_1}^g \theta_{i1} + \lambda_{4\theta_2}^g \theta_{i2} + v_{ip}\right) * \\
& \sigma_g^{-1} f\left(\sigma_g^{-1}\left[grds_i - \beta_{50}^g - \beta_{5w} W_i - \lambda_{5\theta_1}^g \theta_{i1} - \lambda_{5\theta_2}^g \theta_{i2} - \lambda_{5v_p}^g v_{ip}\right]\right) * \\
& F\left(\beta_{60}^g + \beta_{6w} W_i + \lambda_{6\theta_2}^g \theta_{i2} + \lambda_{6v_p}^g v_{ip}\right) * \\
& F\left(\beta_{70}^g + \beta_{7z} Z_i + \lambda_{7\theta_1}^g \theta_{i1} + \lambda_{7\theta_2}^g \theta_{i2} + \lambda_{7v_p}^g v_{ip}\right)
\end{aligned} \tag{9}$$

This example contribution is for an individual, of gender g, who attended university, had university aspirations, scored in the bottom PISA quartile, always turned in their homework on time,

[6]The coefficients on the observed variables are constrained to be the same for both genders in all of the equations. Allowing these to vary in a model where the factor locations are also the same makes the model computationally infeasible. I have, however, also estimated the model separately by gender, which allows every parameter to differ. With these models it is not possible to compare the scale of the factors. However, I can gauge whether differences in the coefficients on the observed variables contribute substantially to the gender gap. Generally, the role they play is relatively small and the rest of the results change little. These results are available by request.

and never just wanted to get by. This individual's parent also had university level aspirations for their child and saved for their child's education.

The $F()$'s are cumulative normal distribution functions; $f()$ is the normal pdf; the $p(.)$'s are probabilities associated with the points of support; and, Z is a vector of observed variables which includes, parental education, family income, indicators for rural residence, immigrant status, and living in a two-parent family. This vector also includes province dummies and distance from the students' high school to the nearest university. The W vector includes all the variables in Z except the distance to nearest university, and it enters equations where the costs of university are unlikely to directly affect the outcome.

With the exception of the $PISA$ and $grades$ equations, each component of the likelihood is a Probit. Grades enter linearly and the PISA quartiles are modeled as an ordered Probit, where $PISA_1$ is the cut-off value between the first and second quartile. The standard deviations of PISA and grades are σ_p and σ_g, respectively.

4 Results

Before discussing the results from the factor model, I begin with results from Probit regressions where the measurement variables act as proxies for the underlying factors. These simple models describe key patterns and correlations in the data. Although my specifications differ somewhat from Frenette and Zeman (2007), these regressions also permit a comparison between the outcomes at age 19 (Cycle 3) and age 21 (Cycle 4). The marginal effects from these regressions are presented in Table 3, separately by gender. The dependent variable in each case is the dummy variable that equals one if the youth attended university by age 21 and zero otherwise. The standard errors are clustered by school because the first stage of sampling was school-based.

The first set of regressions include only socio-economic and household characteristics. These characteristics include a set of six dummy variables describing the parents' highest level of education. The reference category is 'both parents did not finish high school'. The remaining categories are: Both parents have a Bachelors degree or higher, one parent has a Bachelors or higher, both parents have a post-secondary education (PSE) credential that is not a university degree, only one parent has PSE below the Bachelors level, both parents have a high school diploma, and one parent has a high school diploma. Lone parent families are coded into the 'both parents' categories.

The set of socioeconomic variables also includes indicators for whether the youth lives in a two parent family, in a rural area, is an immigrant, or is Indigenous. The natural log of family income is also included. I adjust family income to account for household size by dividing total before-tax family income by the square root of the number of household members. All of these family background characteristics are reported by the parents.

The final variable included in this set is the log minimum distance to the nearest university from the youths' high school.[7] This measure, which captures some of the costs associated with university participation, is calculated as a straight-line distance based on the latitude and longitude of the high school and nearest university. The latitude and longitudes are found by matching the university and school postal codes using the 2001 Postal Code Conversion Files (PCCF).[8]

Parental education is the strongest predictor of university participation, a result that is commonly found in this and other Canadian data sets (Drolet, 2005; Christofides et al., 2009; Finnie and Wismer, 2011). Youth whose parents both have a university degree are more than 40 percentage points more likely to attend university than similar youth whose parents were both high school dropouts. Family income also predicts participation but the impacts are smaller in magnitude.

Although educational attainment among Indigenous Canadians is typically much lower than among other Canadians, the Indigenous indicator is not statistically significant in these regres-

[7]Only the location of the high school is available in the YITS without special permission. In many cases, youth will live relatively close to their high school, however, this may not always be the case, possibly generating measurement error.

[8]The university postal codes were graciously given to me by Marc Frenette, who collected them from the Association of Universities and Colleges, Canada (AUCC) website. Any errors in their use are mine alone.

sions. This is partly because there are relatively few in the sample, and as such the standard errors are large. Additionally, the effect may be underestimated since schools on First Nations reserves and in the Territories were not sampled.

There are two notable gender differences found in the effect of family structure and distance from the nearest university. Girls benefit, in terms of participation, from living in a two parent family much more than boys do. Frenette and Zeman (2007) use more categories of family structure, and they find that the negative impact of living in a family without biological parents is greater for girls, which is consistent with what I find. The second difference is the impact of distance to nearest university, which is very small and statistically insignificant for girls. Frenette (2004), who shows that distance predicts university participation, particularly for low-income families, does not consider the impact for boys and girls separately. Although Frenette and Zeman (2007) do not include a measure of distance in their analysis, they similarly find that wage premiums are uncorrelated with girls' participation, after controlling for a wide variety of factor.[9]

The second set of regressions in Table 3 introduce the measurement variables, which can be thought of as proxies for the underlying factors here. For both boys and girls, the effect of the socioeconomic variables falls in size and statistical significance when the proxy variables are added. This point is emphasized in FGG, where we show that the parental-education gradient in dropping out can be explained by the three underlying factors: cognitive and non-cognitive skills, and parental valuations of education. Furthermore in these regression, each of the measurements strongly predict dropping out, except the indicator for whether the child never 'just wants to get by'. This variable covaries with grades and the 'always does homework on time' variable, such that when those variables are not included in the regression, the 'getby' variable is statistically significant.

In the third set of regressions, I include school characteristics, a measure of peers and an

[9] Frenette and Zeman (2007) use geographic variation in wage premiums. It could be that girls are less responsive to local labour markets, rather than unresponsive to wage differences altogether.

indicator for whether the gender of the responding parent coincides with the child's gender. The school characteristics include an index of school quality reported by the high school administrator, the ratio of students to teachers, and the ratio of boys to girls. None of these are statistically significant. There is evidence from U.S. data that students are more favourably evaluated by teachers who are similar in terms of gender and race (Dee, 2005). Unfortunately, the YITS does not contain specific information about the characteristics of each student's teachers.

There is also evidence that one's peers, and in particular the gender of those peers, affects achievement in high school (Hoxby, 2000; Hill, 2015). Although the type of information needed to identify peer effects is not available in the YITS, the youth are asked about their closest friends. I include an indicator for whether the youth said that all of their friends 'think completing high school is very important?'. Again, this variable is not statistically significant and the effect size is quite small. Finally, an indicator for whether the responding parent had the same gender as their child is also insignificant.

Overall, the strength of association between participation and the measurement variables can be contrasted with that of the variables included only in the third set of regressions. I do not use this set of variables as measurements or covariates in the factor model primarily because they do not predict university participation after controlling for family background and the measurements which are included in the factor model.

4.1 Factor Model

In this section, I discuss the results from the factor model introduced in Section 3, beginning with the support of the factor distributions. The number of points of support for each factor was determined empirically. The model I present here has three points of support for the cognitive and non-cognitive factors and two points of support for the VPE factor. I began with two points of support and added a third point to each factor in turn. Both the Akaike Information Criterion and the Baysian Information Criterion rejected the model with three cognitive, three VPE and two non-cognitive points of support in favour of the model presented here.[10]

The parameter estimates from the university outcome equation are reported in Table 4, with the constrained factor model presented beside the flexible model, in which the factor distributions and loadings vary by sex. Comparing the two models, the coefficients on the observable variables in the university equation are quite similar, except for the female dummy, which shifts the participation intercept. Relative to the constrained model, the female dummy in the flexible model is about a third of the size and is no longer statistically significant. As I will discuss later, this is the first indication that the factors play a large role in explaining the gender gap in participation.

For the measurement equations, I report the intercepts and factor loads in Table 5. At the bottom of this table, the sum of the log likelihoods for each model are shown. Using these to construct a likelihood ratio test, I can reject the null hypothesis that the factor loadings and distributions are the same for boys and girls with a very high level of significance.[11]

For each of the factors, the loading parameters in the university outcome equation are statistically significant in both models. The size of the factor loadings are difficult to interpret on their own because the scale of any factor is determined by the scale of the measurement equation in which the load is normalized. Nonetheless, the statistical significance implies that each factor plays a role in determining university participation. Similarly, the factor loadings are all statistically significant in the children's aspirations equation. While the PVE factor is related to parental aspirations by construction, the cognitive and non-cognitive factors are also significantly related to parental aspirations. Taken together, these results imply that the factors represent possible channels through which children's and parents' aspirations impact university participation. Moreover, because they jointly determine aspirations and the outcome, the factors reflect characteristics which, although not directly observed in data, are known to parents and their children in grade 10 when the aspirations information was collected.

[10] The model with three points of support in each factor, which is 27 different intercepts, never converged.

[11] The likelihood ratio test statistic is 318.43.

Whether the PVE factor can be interpreted as reflecting parents' views about education, rather than just reflecting children's ability and motivation, is an important issue. The *saved* equation is the second measurement that is taken from the parents' survey. In this equation, the PVE factor loading is statistically significant in both models. In contrast, the cognitive factor loadings are statistically insignificant and very small. This suggests that parental behaviour is more than just another measure of children's ability.

To investigate the size of the factors' impact on university participation, for both genders, I plot a predicted probability evaluated for 12 of the different factor levels in Figure 3. I omit the lowest category of non-cognitive skills because only 5% of girls and 9% of boys fall into this category.[12] Within each graph, the cognitive gradient is observed. Within each row, the different graphs reflect levels of the non-cognitive factor and comparing the bottom and top rows reveals the impact of the PVE factor. I also report each marginal effect in Tables 6 to 8.

Before comparing across genders, there are several general observations that can be made about the predicted probabilities. First, all three factors have a large impact on university participation. The cognitive skill gradient is larger than .5 in all cases. In general, the effect that the non-cognitive factor has on university participation is not quite as large. However, since the cognitive factor also capture the types of non-cognitive skills that are associated with test-taking effort, the non-cognitive effects captured in Figure 3 is a lower bound, and as such may be viewed as quite substantial.

The size of the non-cognitive effects also depend on the level of the PVE factor. Taking as an example the probabilities predicted at the high cognitive level, when the PVE factor is low, the impact of having high non-cognitive skills–relative to the medium level– is 17 and 15 percentage points for boys and girls, respectively. In contrast, the same comparison when the PVE factor is low is 7.8 for boys and 7.2 percentage points for girls. Part of the reason for this is that the probability is already quite high when the PVE factor is high, as such there is not as much scope

[12] A figure with all 18 levels is reported in an appendix.

for non-cognitive skills to improve the chances of attending university.

Similarly, the effect of the PVE factor varies depending on the level of the skills factors at which the impact is evaluated. The impact is largest at the medium level of the non-cognitive factor, ranging between .38 and .40. One way to understand the magnitude of the PVE factor effect is to compare the predicted probabilities in Figure 3 to the unconditional probabilities, which are .38 for boys, and .53 for girls. For girls, if their parent has a low valuation, their probability of attending university is at or below average unless they have both high cognitive and non-cognitive skills. For boys, the low-valuation disadvantage, relative to the average probability, disappears with a high level of one skill and a medium level in the other.

Although the impacts vary, the PVE factor increases the probability of attending university by a large margin for all the levels of skills. For children with high levels of both skills, having a parent who values education raises the chance of attending university from roughly .7 to .94. This is very different from what we found in FGG when we studied the impact of parental valuations on the high school dropout decision. There, we found that the PVE had essentially no effect on children with high cognitive skills. This is because the vast majority of students in the YITS finish high school and having high cognitive skills is enough to virtually guarantee completion.[13] In contrast, taking boys and girls together fewer than half attend university (45%). For the university participation decision, while having high levels of cognitive and non-cognitive skills increases one's chances, there is still plenty of room for factors such as parents' valuation of education to make a difference.

Turning now to a comparison across genders, the most striking result is how small the gender differences really are, once one conditions on the three factors. Among those with high cognitive skills, there is virtually no difference between boys and girls in the probability of attending university, no matter the level of the other factors. The gender participation gap is also far less

[13] It is worth pointing out that the YITS sampling strategy did not include high schools where one might expect to find very high levels of socioeconomic disadvantage such as schools on First Nations reserves and in the Territories.

pronounced at the low level of the PVE factor.

In contrast, a gender gap persist for youth with the low level of cognitive skills. That gender gap is also wider for families with high parental valuations. Girls are about 12 percentage points more likely to attend university than boys if they have low cognitive skills and high levels of non-cognitive and PVE factors. That same difference is 11 percentage points for the medium level of non-cognitive skills. Although these are quite large differences, which compare to the unconditional gap of .15, a relatively small fraction of the mass is estimated at those factor levels. Across all three levels of non-cognitive skills, roughly 9 per cent of both boys and girls have low cognitive skills and a high PVE. Additionally, because these predicted probabilities are the average of a non-linear function evaluated at a particular vector, the difference in any two probabilities stems partly from the concavity of the cumulative normal function. The differences will be more pronounced at lower levels where the function is more convex.

Since, after conditioning on the factors, the predicted probabilities of attending university are very similar, this implies different distributions of the factors for boys and girls. Figure 2 reveals that this is indeed the case. The top panel of Figure 2 shows the marginal distributions for each of the three factors. The bottom panel reports the joint distributions, which are the product of the marginal distributions, since the factors are orthogonal.

Girls have higher average levels of each factor. They are more likely to have the higher level and less likely to have the lower level of cognitive skills. The predicted probability of having the highest level of non-cognitive skills is 13 percentage points higher for girls. Girls are also 13 percentage points more likely to have parents with the highest valuation.

4.2 How much of the gap is explained by the factors?

To further explore the role each factor and factor loading plays in explaining the gender gap in participation, I perform a decomposition exercise, following in the spirit of a Oaxaca-Blinder decomposition. I begin by expressing the unconditional gender gap in terms of the factor model. Unlike with a linear decomposition, it is important to take the average predicted probabilities, rather than evaluate the probability at the average (Fairlie, 1999; Fortin et al., 2011). If F is the cumulative normal distribution, using the participation index in equation (1), the total gender gap is:

$$\begin{aligned}\Delta_U &= U\left(X^f, f\right) - U\left(X^m, m\right) \qquad (10)\\ &= \frac{1}{n^f}\sum_{i=1}^{n^f}\sum_{\theta_1}\sum_{\theta_2}\sum_{v_p} p^f\left(\theta_1\right)p^f\left(\theta_2\right)p^f\left(v_p\right)F\left(\gamma_0^f + \gamma_x X_i^f + \lambda_{0\theta_1}^f\theta_{i1} + \lambda_{0\theta_2}^f\theta_{i2} + \lambda_{0v_p}^f v_{ip}\right)\\ &- \frac{1}{n^m}\sum_{i=1}^{n^m}\sum_{\theta_1}\sum_{\theta_2}\sum_{v_p} p^m\left(\theta_1\right)p^m\left(\theta_2\right)p^m\left(v_p\right)F\left(\gamma_0^m + \gamma_x X_i^m + \lambda_{0\theta_1}^m\theta_{i1} + \lambda_{0\theta_2}^m\theta_{i2} + \lambda_{0v_p}^m v_{ip}\right)\end{aligned}$$

To calculate how much of that raw difference can be attributed to differences in the observed characteristics, I need to evaluate the predicted probabilities using the parameters from one gender and the X-vector from the other gender. Specifically:

$$U\left(X^g, h\right) = \frac{1}{n^g}\sum_{i=1}^{n^g}\sum_{\theta_1}\sum_{\theta_2}\sum_{v_p} p^h\left(\theta_1\right)p^h\left(\theta_2\right)p^h\left(v_p\right)F\left(\gamma_0^h + \gamma_x X_i^g + \lambda_{0\theta_1}^h\theta_{i1} + \lambda_{0\theta_2}^h\theta_{i2} + \lambda_{0v_p}^h v_{ip}\right)$$

Putting these terms together, yields the explained and unexplained differences from the familiar Oaxaca-Blinder decomposition, which can be implemented in two different ways[14]:

Using the male parameters:

$$\Delta_{X^m} = \underbrace{U\left(X^f, m\right) - U\left(X^m, m\right)}_{\text{Explained}} + \underbrace{U\left(X^f, f\right) - U\left(X^f, m\right)}_{\text{Unexplained}} \qquad (11)$$

Using the female parameters:

$$\Delta_{X^f} = \underbrace{U\left(X^f, f\right) - U\left(X^m, f\right)}_{\text{Explained}} + \underbrace{U\left(X^m, f\right) - U\left(X^m, m\right)}_{\text{Unexplained}} \qquad (12)$$

[14] A third decomposition is possible using parameters from the constrained or 'pooled' model. The results from such a decomposition are available from the author. In practice, in this particular example, it makes very little difference to the results which method is used.

The first term, in both (11) and (12), represents the part of the gender gap that can be explained by differences in the observed characteristics, and the second term represents the gap that is 'unexplained'. This decomposition is presented in the first row of Table 9.[15] The set of socio-economic variables included in the university participation index explain essentially none of the total gender gap, which the model estimates to be .1496. This conclusion does not depend on which set of parameters is used in the decomposition. This result is seemingly quite different from Frenette and Zeman (2007) who conclude that socioeconomic characteristics explain about three quarters of the gap. However, their set of socioeconomic characteristics includes variables such as parental aspirations and grades. The goal here is to try to disentangle the different factors that are reflected in those variables. In Frenette and Zeman (2007), variables such as parental education and income also explain very little of the gender participation gap.

Because the flexible factor model allows the distribution of factors and their loadings to vary by gender, I can also perform a decomposition based on the factors. In that case the counterfactual is:

$$U\left(X^g, \gamma_0^g, h\right) = \frac{1}{n^g} \sum_{i=1}^{n^g} \sum_{\theta_1} \sum_{\theta_2} \sum_{v_p} p^h\left(\theta_1\right) p^h\left(\theta_2\right) p^h\left(v_p\right) F\left(\gamma_0^g + \gamma_x X_i^g + \lambda_{0\theta_1}^h \theta_{i1} + \lambda_{0\theta_2}^h \theta_{i2} + \lambda_{0v_p}^h v_{ip}\right)$$

The fraction of the gap attributable to the factor structure is given by this decomposition:

[15] The standard errors are obtained by bootstrapping. In each of 200 repetitions, I randomly draw a sample from the data and a vector of parameters from the estimated sampling distribution.

Using the male parameters: (13)

$$\Delta_{\Lambda\Theta^m} = \underbrace{U\left(X^f, f\right) - U\left(X^f, \gamma_0^f, m\right)}_{\text{Explained by factor structure}} + \underbrace{U\left(X^f, \gamma_0^f, m\right) - U\left(X^m, m\right)}_{\text{Unexplained}}$$

Using the female parameters: (14)

$$\Delta_{\Lambda\Theta^f} = \underbrace{U\left(X^m, \gamma_0^m, f\right) - U\left(X^m, m\right)}_{\text{Explained by factor structure}} + \underbrace{U\left(X^f, f\right) - U\left(X^m, \gamma_0^m, f\right)}_{\text{Unexplained}}$$

Here, the unexplained portion is driven by the female intercept, and the very small differences in the X vector. The portion explained by the factor structure can be further decomposed into components explained by each factor, or the factor loading. For example, to learn how much of the gap occurs because girls are more likely to have a higher level of the cognitive factor, I can construct the following counterfactual predicted probability:

$$U\left(g, p^h\left(\theta_1\right)\right) = \frac{1}{n^g}\sum_{i=1}^{n^g}\sum_{\theta_1}\sum_{\theta_2}\sum_{v_p} p^h\left(\theta_1\right) p^g\left(\theta_2\right) p^g\left(v_p\right) F\left(\gamma_0^g + \gamma_x X_i^g + \lambda_{0\theta_1}^g \theta_{i1} + \lambda_{0\theta_2}^g \theta_{i2} + \lambda_{0v_p}^g v_{ip}\right)$$

Then, I can calculate the part of the gap attributed to the distribution of θ_1 with:

Using the male parameters: (15)

$$U\left(X^f, f\right) - U\left(X^f, \gamma_0^f, m\right) = \underbrace{U\left(X^f, f\right) - U\left(f, p^m\left(\theta_1\right)\right)}_{\text{Explained by } p^m(\theta_1)} + \underbrace{U\left(f, p^m\left(\theta_1\right)\right) - U\left(X^f, \gamma_0^f, m\right)}_{\text{Unexplained}}$$

Using the female parameters:

$$U\left(X^m, \gamma_0^m, f\right) - U\left(X^m, m\right) = \underbrace{U\left(m, p^f\left(\theta_1\right)\right) - U\left(X^m, m\right)}_{\text{Explained by } p^f(\theta_1)} + \underbrace{U\left(X^m, \gamma_0^m, f\right) - U\left(m, p^f\left(\theta_1\right)\right)}_{\text{Unexplained}}$$

Performing a detailed decomposition, that is assigning a portion of the gap to each factor and its loading, poses some problems because the model is non-linear.[16] In particular, if performed sequentially, the order in which I perform the decomposition can affect the results. Alternatively,

[16]See Fortin et al. (2011) for a detailed discussion of the issues associated with non-linear decompositions.

if the decomposition is conducted piecewise, the components of the decomposition will not necessarily add up to the whole. Because the second approach is arguably more transparent, I have chosen to perform the decomposition by switching one factor or one loading at a time. I also perform a decomposition where I switch both the loading and probability, to calculate the total impact of the factor. The equations describing each decomposition, which take an analogous form to (15), are presented in an appendix.

As was the case with the observed characteristics, the factor decompositions, also reported in Table 9, are very similar whether I use the female or male coefficients to build the counterfactual. The factor structure, taken as a whole, accounts for .1323 of the total gap of .1496, or 88 percent. I begin the detailed decompositions by considering separately the net effect of each factor. For example, Row (3) illustrates how much of the gap occurs because girls are more likely to have higher cognitive skills and because the impact of those skills is larger. Rows (6) through (8) in Table 9 investigates how much of that is driven by differences in factor loadings alone, while the remaining rows, (9) through (11), report the part driven by the factor distributions. For the PVE and non-cognitive factors, the factor loadings play virtually no role, while for the cognitive factor both the distribution and the loadings matter.

Rows (3) through (5) suggest that each of the factors plays an important role.[17] The cognitive skills factor accounts for the largest share, 40%, of the participation gap, which is .06 percentage points. The part of the gap attributable to non-cognitive skills is smaller at .03. That the non-cognitive factor plays a smaller role is again not surprising since the cognitive factor will absorb any of the soft skills associated with 'test-taking' effort.

The parental valuation of education factor also explains roughly 28 per cent of the gender gap in participation.[18] This implies that parents' play a much larger role than what one might con-

[17]The decompositions in Rows (3) through (5) are less precisely estimated than the others because the counterfactuals involve switching more parameters, and hence include the variability of more parameters. Thus, although the 95% confidence interval around the decompositions in Rows (4) and (5) includes zero, they do not in Rows (10) and (11).

[18]Although, because of the non-linearities, the shares do not add up exactly to the whole explained portion, the discrepancy is quite small in practice.

clude from considering parental aspirations alone. In the simple Oaxaca-Blinder decompositions in Frenette and Zeman (2007), the parental aspirations variable, on its own, accounts for less than 10 per cent of the gap measured in the third cycle of YITS. However, parent's also influence university participation through their children's homework effort and grades. The factor model makes it possible to quantify, and aggregate into the PVE factor, the different channels through which parents' valuations operate.

4.3 Investigating why parental valuations are higher for girls?

The interpretation of the PVE factor as a measure of how much parents value education hinges on the restrictions imposed in the model. Key among those is the assumption that PISA reading scores are not a function of parental valuations. If there is a dimension of cognitive skill which is orthogonal to PISA reading skills but correlated with the parental measurement equations, then the PVE factor might simply reflect that unobserved ability. That would further imply that girls have higher PVE levels simply because they are more skilled in the unobserved dimension.

In the YITS data, there are two other test scores which I use to investigate this possibility. A random subset of the students, roughly half, wrote the PISA science test, while another subset wrote the PISA math test. If, after controlling for the reading scores, the PVE factor is correlated with the math or science scores, this would imply there is an important omitted skill biasing the PVE factor. I extract an estimated PVE factor for each sample member using Bayes Rule[19]:

$$\hat{\Theta} = \int \frac{p(Y|\hat{\Theta}, X, Z; \hat{\Gamma})p(\hat{\Theta}|X, Z; \hat{\Gamma})}{p(Y|X, Z)} d\hat{\Theta} \tag{16}$$

where Y is a matrix of the participation outcome and all of the measurements, $\hat{\Gamma}$ is all of the estimated parameters in the model, and $\hat{\Theta}$ is the vector of the three estimated factors.

In Table 10, I report the results from a regression of Math and Science scores on the estimated PVE factor and the reading test scores. Unsurprisingly, the reading scores are highly correlated

[19] We also performed a similar exercise in FGG.

with both math and science scores. However, after controlling for those scores, the remaining variation in Science and Math scores is not statistically significantly related to the parental valuation factor. Indeed, the estimated coefficient is negative. This evidence supports the claim that the PVE factor is not just another measure of skills.

The key findings are also robust to different normalizations. Very little changes if I normalize the factor loading in the 'getting by' equation (7), instead of the homework measurement (Equation 4). I also estimate a version of the model where the cognitive skills factor enters equations (4) and (7) instead of the PVE factor. Again, very little changes either quantitatively or qualitatively.[20]

If unobserved ability is not driving the results, why, then, are girls' parents more likely to highly value their daughters' education? After answering the aspirations question, parents were asked a follow-up question: "What is the main reason you hope your child will get this level of education?"[21] In Table 11, I present the distribution of answers given, separately for boys and girls and whether the parent had indicated 'university' or 'less than university' aspirations.

Although not directly informative about the unobserved PVE factor, this information does shed some light on what the parents had in mind while answering the aspirations question. Because the PVE factor loading is normalized to one in the parental aspirations question, 'university' aspirations means parents have a higher PVE. As such, when a reason is relatively more common among those with university aspirations, that reason will be correlated with higher parental valuations.

By far the most common reason given is 'better job opportunities or pay', but it is relatively more common among boys' parents and those whose aspirations were less than university. As such, this reason is unlikely to be driving the gender difference in parental valuations. While earnings premiums and job opportunities have played a larger role in explaining the U.S. gender

[20] These estimates are not shown but are available upon request.

[21] In addition to the answers listed in the table, 'best choice in terms of financial costs' was also listed as an option. I included this choice in the 'other' category because it was chosen by very few parents. In the original full sample of 26,063, only 223 parents indicated costs were the main reason.

gap (Jacob, 2002; Fortin et al., 2015), Frenette and Zeman (2007) attribute only 5% of the gender gap to differences in the returns to university. They further point out that earnings premiums are not correlated with girls' participation.[22]

Only two reasons are more prevalent among the parents' with university aspirations. The first of these is 'Best match with child's ability', however, this reason is marginally more common among boys' parents. 'Valuable for personal growth and learning', is the single reason which is both more common among girls' parents and correlated with a higher valuation of education. Among those with university aspirations for their children, the parents of girls were 2.55 percentage points more likely to give this reason.

While it is impossible to know what parents took this phrase to mean precisely, it certainly points toward the non-pecuniary benefits of education. Discussion of such benefits has entered the economic literature relatively recently, yet, their existence and importance has empirical support (Oreopoulos and Salvanes, 2011). A key way in which non-pecuniary benefits may differ across gender is through the marriage market.

Schooling has value in the marriage market because it helps attract a desirable match, and because it can increase one's well-being within a marriage (Becker, 1973; Goldin, 1992; Peters and Siow, 2000). Chiappori et al. (2009) and Chiappori et al. (forthcoming) show that the returns to education in the marriage market can be higher for women when technological advancements reduce the time needed for home production, and as investment in children's human capital becomes more important. Echevarria and Merlo (1999) link gender differences in education to parental investments in an intergenerational household bargaining model. Altruistic parents make investments in their children's education, conditional on gender, taking into account the potential returns to education within a future marriage. Although their model still predicts, higher levels of education for boys, the difference is smaller than a pure Beckerian-investment

[22] It is worth pointing out that this evidence is not necessarily inconsistent with findings in the literature that wage premiums are responsible for the *trend* in girls' university participation and graduation rates. The changes over time in labour market conditions for women are qualitatively and quantitatively different than the variation found within a single cohort.

model would predict.

Non-pecuniary benefits, which might include higher returns to education though the marriage market, are of course, not the only possible explanation for the higher PVE factor. The question asking parents about the reasons for their aspirations, asked only about one reason, the most important reason. Other reasons could very well play prominent roles in explaining their aspirations. On the whole, however, there is evidence to support the claim that the differences in the parental valuations are not merely a reflection of ability.

5 Conclusion

In the Youth in Transition Survey, 53% of girls and 38% of boys had ever attended university by age 21. I have sought to identify and quantify the underlying factors that contribute to that gender gap in university participation among Canadian youth. Using the factor model from Foley, Gallipoli, and Green (2014), I focus on three factors linked to cognitive skills, non-cognitive skills and parental valuations of education (PVE). I find that all three factors have large impacts on university participation. The cognitive skill gradient is very large, raising the probability of attending university by roughly .5. The impact of non-cognitive skills and parental valuations can be almost as large but the impact depends on the level of the other factors. The impact of non-cognitive factors is larger when the parental valuation factor is lower. Similarly, the impact of the PVE factor is larger among less skilled youth.

All three factors also play an important role in explaining the gender gap in university participation. At the highest cognitive skill level, the probability of attending university is virtually identical for boys and girls. Although gaps do persist among less skilled youth, overall the factor structure can account for 88 per cent of the total 15 percentage point gap. This is primarily because girls have higher levels of all three factors. The cognitive skill factor explains the largest fraction, but non-cognitive skills and parental valuations also play important roles. Indeed, the PVE factor explains 28 percent of the gap.

The results in this paper suggest that parents play a much larger role than one might expect from simply considering parents' education or their stated aspirations for their children. This is partly because the factor structure accounts for the extent to which variables, such as grades, are influenced not only by youth's skills and motivations, but also by how much their parents value education.

While contributing to a fuller understanding of why girls and boys differ in their propensity to attend university, this evidence also generates information that can be used when thinking about programs to promote university attendance, particularly among young men. As Fortin et al. (2015) point out, interventions such as 'Future to Discover' that offer information and financial assistance in the early years of high school show the potential to shift boys' plans for the future. My results also point toward models like the 'Future to Discover' intervention because it involved parents in the information component (Ford and Kwakye, 2016).

Involving parents is potentially important for two reasons. First, there is evidence that boys are less likely to make use of services (Angrist et al., 2009). Parental involvement might encourage boys to take advantage of available resources. The second reason goes beyond gender differences, to highlight the overall impact of parental valuations. To the extent that the parental valuation of education can be interpreted as something that is separable from ability and skills, these results suggest that parents' play a contemporaneous role in the university participation decision.

This was a key point that was emphasized in FGG in the context of the high school dropout decision. That point is worth restating here because of a critical difference between dropping out of high school and attending university. Among those with high cognitive skills, virtually everybody finishes high school in the YITS data. As such, in that group, parental valuations have no impact on dropping out. The same is not true for university participation. Even among the group of students with the highest cognitive and non-cognitive skills, parental valuations still have a large impact on the probability of attending university. Insofar as it is socially and economically desirable to encourage university participation among those with the highest level of skills, this is an issue that should be of relevance to policy makers.

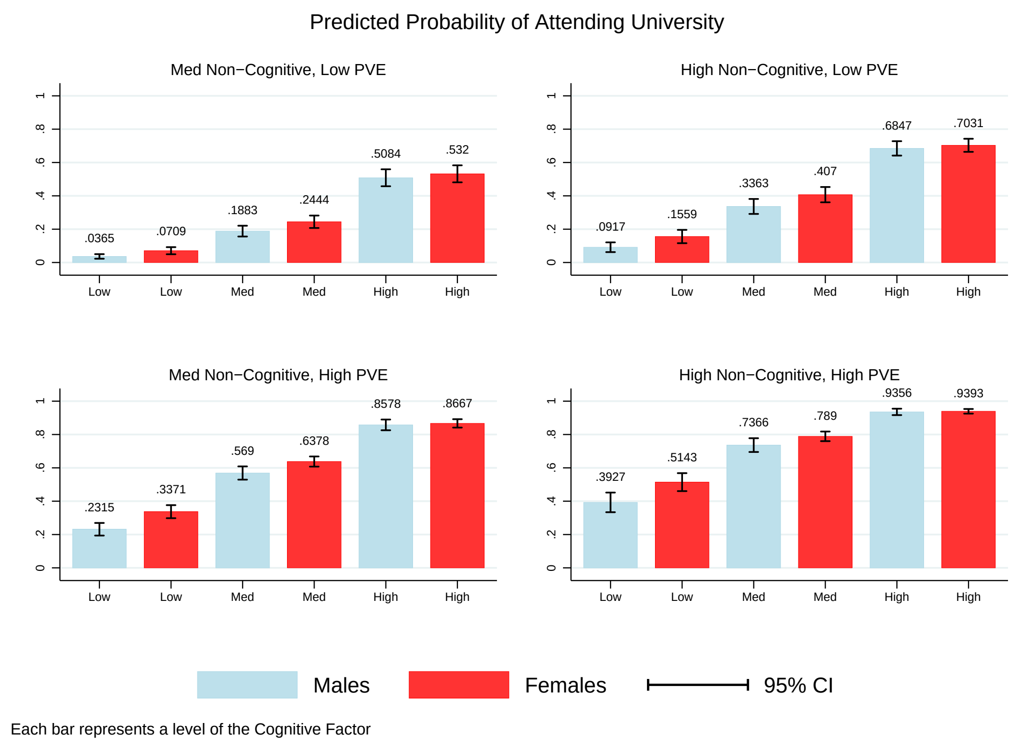

Figure 1: Predicted Probability of Attending University evaluated at each level of the estimated factors.

Notes: Confidence intervals constructed with standard errors estimated using the Delta Method and numerical derivatives.

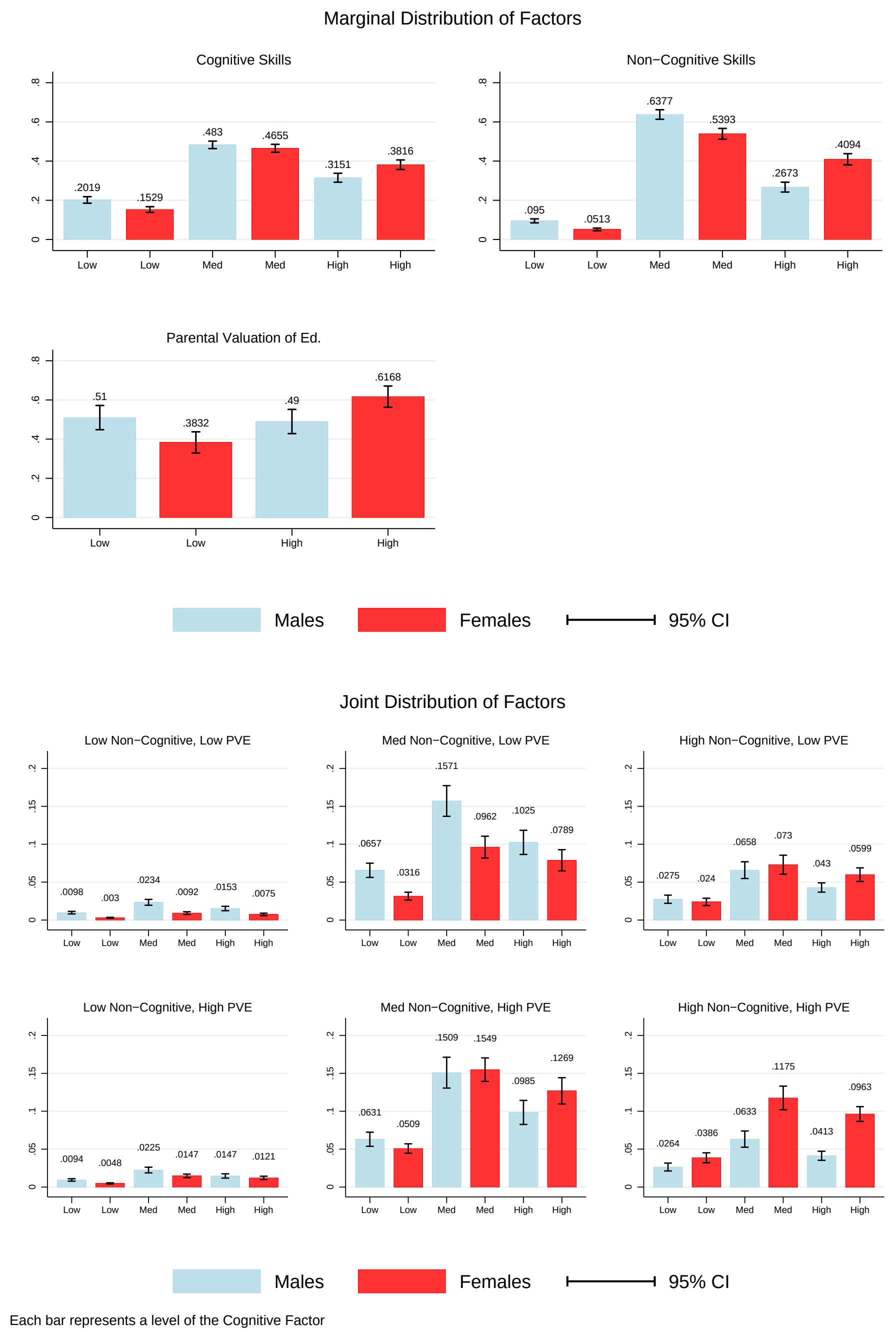

Figure 2: Estimated Factor Distributions

Notes: Confidence intervals constructed with standard errors estimated using the Delta Method and numerical derivatives.

Table 1: Outcome and Measurement Variable Means and Standard Deviations (in parentheses)

	Pooled	**Males**	**Females**
University participation	0.4569	0.3797	0.5270
	(0.4982)	(0.4854)	(0.4993)
Child wants university degree	0.6791	0.6239	0.7292
	(0.4669)	(0.4845)	(0.4444)
Parent hopes child gets degree	0.6839	0.6454	0.7189
	(0.4650)	(0.4784)	(0.4496)
PISA quartiles			
Quartile 1 (bottom)	0.1646	0.2194	0.1148
	(0.3709)	(0.4139)	(0.3188)
Quartile 2	0.2209	0.2335	0.2094
	(0.4149)	(0.4231)	(0.4069)
Quartile 3	0.2961	0.2911	0.3007
	(0.4566)	(0.4543)	(0.4586)
Quartile 4 (top)	0.3183	0.2560	0.3751
	(0.4659)	(0.4364)	(0.4842)
Overall grades (Percent)	77.2113	75.3619	78.8929
	(10.2311)	(10.3449)	(9.8300)
Child never just wants to get by	0.3298	0.2334	0.4174
	(0.4702)	(0.4230)	(0.4932)
Always does homework on time	0.2599	0.2007	0.3138
	(0.4386)	(0.4006)	(0.4641)
Parent saved for child's education	0.6329	0.6391	0.6274
	(0.4820)	(0.4803)	(0.4836)
Sample Size	9810	4507	5303

Table 2: Socioeconomic Variable Means and Standard Deviations (in parentheses)

	Pooled	**Males**	**Females**
Female	0.5406		
	(0.4984)		
Lives in two parent family	0.7548	0.7674	0.7434
	(0.4302)	(0.4225)	(0.4368)
Indigenous	0.0257	0.0268	0.0247
	(0.1583)	(0.1617)	(0.1552)
Immigrant	0.0690	0.0625	0.0750
	(0.2535)	(0.2420)	(0.2635)
Rural	0.2443	0.2390	0.2491
	(0.4297)	(0.4265)	(0.4325)
Minimum distance to Uni (Km)	42.5562	42.2181	42.8637
	(65.6158)	(65.2394)	(65.9611)
Adult Equivalent Family Income (1000)	36.3717	37.3733	35.4611
	(27.7246)	(29.7114)	(25.7547)
Ln of family income	10.3361	10.3682	10.3068
	(0.5856)	(0.5676)	(0.6001)
Ln minimum distance to Uni	2.8221	2.8117	2.8316
	(1.5213)	(1.5278)	(1.5154)
Highest level of parental education			
Both parents have less than HS	0.0452	0.0412	0.0488
	(0.2078)	(0.1988)	(0.2155)
One parent has HS	0.0539	0.0527	0.0549
	(0.2257)	(0.2234)	(0.2279)
Both parents have HS	0.1388	0.1428	0.1352
	(0.3458)	(0.3499)	(0.3419)
One parent has PSE below BA	0.2108	0.2052	0.2158
	(0.4079)	(0.4039)	(0.4114)
Both parents have PSE below BA	0.2405	0.2422	0.2389
	(0.4274)	(0.4285)	(0.4264)
One parent has BA or more	0.1633	0.1683	0.1588
	(0.3697)	(0.3742)	(0.3655)
Both parents have BA or more	0.1476	0.1476	0.1476
	(0.3547)	(0.3547)	(0.3547)
Province While in High school			
Newfoundland	0.0246	0.0236	0.0255
	(0.1548)	(0.1517)	(0.1575)
Prince Edward Island	0.0070	0.0068	0.0072
	(0.0833)	(0.0820)	(0.0846)
Nova Scotia	0.0322	0.0326	0.0318
	(0.1765)	(0.1776)	(0.1754)
New Brunswick	0.0312	0.0276	0.0344
	(0.1738)	(0.1639)	(0.1823)
Quebec	0.1617	0.1600	0.1632
	(0.3682)	(0.3666)	(0.3696)
Ontario	0.4154	0.4134	0.4172
	(0.4928)	(0.4925)	(0.4931)
Manitoba	0.0421	0.0437	0.0407
	(0.2009)	(0.2045)	(0.1976)
Saskatchewan	0.0243	0.0251	0.0235
	(0.1539)	(0.1564)	(0.1515)
Alberta	0.1171	0.1182	0.1161
	(0.3215)	(0.3228)	(0.3203)
British Columbia	0.1446	0.1491	0.1405
	(0.3517)	(0.3562)	(0.3476)
Sample Size	9810	4507	5303

Table 3: Observed characteristics and university participation: Marginal Effects from Probit Regressions (Standard Errors in Parenthesis)

	(1) Male	(1) Female	(2) Male	(2) Female	(3) Male	(3) Female
Highest level of parental education–Reference group Both less than HS						
One parent has HS	0.090	0.000	0.035	-0.009	0.034	-0.009
	(0.071)	(0.063)	(0.058)	(0.046)	(0.058)	(0.047)
Both parents have HS	0.169*	0.093	0.063	0.035	0.062	0.035
	(0.066)	(0.049)	(0.056)	(0.039)	(0.055)	(0.039)
One parent has PSE below BA	0.182**	0.103*	0.082	0.040	0.081	0.039
	(0.065)	(0.047)	(0.054)	(0.036)	(0.054)	(0.037)
Both parents have PSE below BA	0.164**	0.240***	0.032	0.118**	0.029	0.115**
	(0.064)	(0.046)	(0.052)	(0.036)	(0.052)	(0.036)
One parent has BA or more	0.373***	0.307***	0.146**	0.138***	0.144**	0.136***
	(0.065)	(0.048)	(0.055)	(0.038)	(0.055)	(0.038)
Both parents have BA or more	0.469***	0.436***	0.182***	0.192***	0.179***	0.190***
	(0.065)	(0.050)	(0.052)	(0.041)	(0.052)	(0.041)
Lives in two parent family	0.044	0.133***	0.000	0.094***	-0.001	0.094***
	(0.024)	(0.022)	(0.020)	(0.019)	(0.020)	(0.019)
Log of family income	0.043*	0.083***	0.014	0.034*	0.015	0.035*
	(0.017)	(0.018)	(0.015)	(0.015)	(0.016)	(0.015)
Log minimum distance to Uni	-0.023**	-0.002	-0.021***	0.001	-0.021***	0.002
	(0.007)	(0.007)	(0.005)	(0.007)	(0.005)	(0.006)
Rural	-0.050	-0.049*	-0.019	-0.036	-0.020	-0.040
	(0.026)	(0.024)	(0.021)	(0.020)	(0.021)	(0.020)
Indigenous	-0.064	-0.104	0.011	-0.014	0.011	-0.010
	(0.058)	(0.054)	(0.052)	(0.043)	(0.051)	(0.043)
Immigrant	0.030	0.038	-0.018	0.011	-0.018	0.011
	(0.047)	(0.044)	(0.035)	(0.036)	(0.035)	(0.037)
PISA Reading Test Scores–Reference group bottom quartile						
Q2 PISA Score			0.063*	0.178***	0.065*	0.179***
			(0.026)	(0.032)	(0.026)	(0.032)
Q3 PISA Score			0.139***	0.208***	0.141***	0.209***
			(0.025)	(0.031)	(0.025)	(0.031)
Q4 PISA Score			0.215***	0.282***	0.217***	0.282***
			(0.026)	(0.032)	(0.026)	(0.032)
Other measurements						
Child never just wants to get by			0.028	-0.002	0.026	-0.004
			(0.017)	(0.017)	(0.017)	(0.017)
Always does homework on time			0.064***	0.053**	0.063**	0.054**
			(0.019)	(0.018)	(0.019)	(0.018)
Child wants university degree			0.149***	0.181***	0.148***	0.176***
			(0.019)	(0.021)	(0.019)	(0.020)
Parent hopes child gets degree			0.095***	0.114***	0.094***	0.115***
			(0.018)	(0.018)	(0.018)	(0.019)
Overall grades			0.011***	0.010***	0.011***	0.010***
			(0.001)	(0.001)	(0.001)	(0.001)
Variables not included in factor model						
Index of school quality					-0.003	0.003
					(0.007)	(0.009)
Ratio of students to teachers in school					-0.001	-0.005*
					(0.002)	(0.002)
Ratio of boys to girls in school					0.068	0.148
					(0.090)	(0.082)
Missing school information					-0.008	-0.038
					(0.059)	(0.061)
All friends think school important					0.018	0.029
					(0.016)	(0.017)
Responding parent same gender as child					0.003	0.025
					(0.018)	(0.020)
Sample Size	4507	5303	4507	5303	4507	5303

Table 4: Coefficients from Factor Models, University Participation (Standard Errors in Parenthesis)

	Restricted Model	Flexible Model Common	Flexible Model Male	Flexible Model Female
Female	0.3636***	0.1045		
	(0.0358)	(0.1929)		
Lives in two parent family	0.1749***	0.1741***		
	(0.0452)	(0.0450)		
Rural	-0.2189***	-0.2195***		
	(0.0466)	(0.0465)		
Immigrant	0.2586**	0.2579**		
	(0.0913)	(0.0913)		
Ln minimum distance to Uni	-0.0514***	-0.0508***		
	(0.0127)	(0.0127)		
Ln of family income	0.2527***	0.2527***		
	(0.0347)	(0.0344)		
Highest level of parental education–Reference group Both less than HS				
One parent has HS	0.8522***	0.8382***		
	(0.1035)	(0.1028)		
Both parents have HS	1.1017***	1.0851***		
	(0.0936)	(0.0929)		
One parent has PSE below BA	1.1264***	1.1094***		
	(0.0889)	(0.0883)		
Both parents have PSE below BA	1.3866***	1.3746***		
	(0.0888)	(0.0879)		
One parent has BA or more	1.7001***	1.6828***		
	(0.0967)	(0.0959)		
Both parents have BA or more	2.0894***	2.0770***		
	(0.1042)	(0.1033)		
Intercept	-5.2151***	-5.0544***		
	(0.3745)	(0.3815)		
Cognitive factor load ($\lambda^g_{0\theta_1}$)	0.0184***		0.0198***	0.0171***
	(0.0008)		(0.0011)	(0.0010)
Non-cognitive factor load ($\lambda^g_{0\theta_2}$)	0.4562***		0.4522***	0.4524***
	(0.0307)		(0.0403)	(0.0404)
PVE factor load ($\lambda^g_{0v_p}$)	1.0064***		0.9920***	0.9924***
	(0.0783)		(0.0934)	(0.0854)

Table 5: Selected Coefficients from Factor Models, Measurement Equations (Standard Errors in Parenthesis)

	Restricted Model		**Flexible Model**	
		Common	**Male**	**Female**
yasp equation				
Intercept	-3.8908***	-3.8665***		
	(0.4059)	(0.4185)		
Cognitive factor load ($\lambda^g_{1\theta_1}$)	0.0144***		0.0150***	0.0150***
	(0.0008)		(0.0010)	(0.0010)
Non-cognitive factor load ($\lambda^g_{1\theta_2}$)	0.3484***		0.3735***	0.3735***
	(0.0300)		(0.0389)	(0.0389)
PVE factor load ($\lambda^g_{1v_p}$)	1.2956***		1.2727***	1.2727***
	(0.0996)		(0.1133)	(0.1133)
PISA equation				
Intercept	479.2876***	477.9310***		
	(13.7654)	(13.7696)		
getby equation				
Intercept	-2.3379***	-2.3067***		
	(0.3009)	(0.3142)		
PVE factor load ($\lambda^g_{3v_p}$)	0.4279***		0.4549***	0.4549***
	(0.0475)		(0.0644)	(0.0644)
parasp equation				
Intercept	-3.8312***	-3.8280***		
	(0.3581)	(0.3641)		
Cognitive factor load ($\lambda^g_{4\theta_1}$)	0.0110***		0.0117***	0.0117***
	(0.0006)		(0.0009)	(0.0009)
Non-cognitive factor load ($\lambda^g_{4\theta_2}$)	0.2478***		0.2898***	0.2898***
	(0.0250)		(0.0332)	(0.0332)
Sample Size	9810	9810	9810	9810

Table 5: Selected Coefficients from Factor Models, Measurement Equations (Standard Errors in Parenthesis) continued

	Restricted Model	**Flexible Model**		
		Common	**Male**	**Female**
grades equation				
Intercept	56.6113***	56.5734***		
	(0.1531)	(0.1550)		
Cognitive factor load ($\lambda^g_{5\theta_1}$)	0.1861***		0.1862***	0.1862***
	(0.0041)		(0.0041)	(0.0041)
Non-cognitive factor load ($\lambda^g_{5\theta_2}$)	8.5007***		8.4543***	8.4543***
	(0.3160)		(0.3147)	(0.3147)
PVE factor load ($\lambda^g_{5v_p}$)	0.0536**		0.0506*	0.0506*
	(0.0163)		(0.0252)	(0.0252)
hmwrk equation				
Intercept				
Non-cognitive factor load ($\lambda^g_{6\theta_2}$)	0.6122***		0.6109***	0.6109***
	(0.0340)		(0.0457)	(0.0457)
PVE factor load ($\lambda^g_{6v_p}$)	0.5144***		0.4765***	0.4765***
	(0.0569)		(0.0739)	(0.0739)
saved equation				
Intercept	-5.0911***	-4.9581***		
	(0.2825)	(0.2870)		
Cognitive factor load ($\lambda^g_{7\theta_1}$)	0.0005		0.0011	0.0011
	(0.0005)		(0.0006)	(0.0006)
Non-cognitive factor load ($\lambda^g_{7\theta_2}$)	0.0690***		0.0472	0.0472
	(0.0188)		(0.0253)	(0.0253)
PVE factor load ($\lambda^g_{5v_p}$)	0.2301***		0.1601**	0.1601**
	(0.0348)		(0.0491)	(0.0491)
Factor' locations				
Low cognitive factor location	-107.4769***	-107.2440***		
	(2.3672)	(2.3678)		
Medium cognitive factor location	-53.6556***	-53.5385***		
	(1.1820)	(1.1821)		
High non-cognitive factor location	2.2485***	2.2638***		
	(0.0835)	(0.0842)		
Medium non-cognitive factor location	3.4320***	3.4571***		
	(0.1275)	(0.1285)		
High PVE factor location	1.2390***	1.2471***		
	(0.0613)	(0.0620)		
Sample Size	9810	9810	9810	9810
Log likelihood	-67993.525	-67993.525		

Table 6: Marginal Effect of Cognitive Factor

	Low Non-Cognitive		Medium Non-Cognitive		High Non-Cognitive	
	Low PVE	High PVE	Low PVE	High PVE	Low PVE	High PVE
Marginal Effect of High to Low						
Boys	-0.1927***	-0.4718***	-0.5930***	-0.5268***	-0.6262***	-0.5429***
	(0.0306)	(0.0249)	(0.0223)	(0.0372)	(0.0238)	(0.0283)
Girls	-0.2064***	-0.4611***	-0.5472***	-0.5037***	-0.5296***	-0.4251***
	(0.0326)	(0.0242)	(0.0223)	(0.0333)	(0.0234)	(0.0267)
Marginal Effect of Medium to Low						
Boys	-0.1572***	-0.3200***	-0.3484***	-0.3390***	-0.2888***	-0.1991***
	(0.0235)	(0.0178)	(0.0165)	(0.0205)	(0.0123)	(0.0141)
Grils	-0.1567***	-0.2876***	-0.2961***	-0.2980***	-0.2289***	-0.1503***
	(0.0231)	(0.0160)	(0.0152)	(0.0167)	(0.0101)	(0.0104)

Table 7: Marginal Effect of Non-Cognitive Factor

	Low Cognitive		Medium Cognitive		High Cognitive	
	Low PVE	High PVE	Low PVE	High PVE	Low PVE	High PVE
Marginal Effect of High to Low						
Boys	-0.0879***	-0.2971***	-0.4882***	-0.3390***	-0.4950***	-0.3551***
	(0.0143)	(0.0226)	(0.0321)	(0.0304)	(0.0350)	(0.0398)
Girls	-0.1463***	-0.3477***	-0.4871***	-0.4154***	-0.4844***	-0.3367***
	(0.0193)	(0.0239)	(0.0331)	(0.0314)	(0.0368)	(0.0403)
Marginal Effect of Medium to Low						
Boys	-0.0551***	-0.1480***	-0.1763***	-0.1612***	-0.1676***	-0.0779***
	(0.0091)	(0.0136)	(0.0148)	(0.0162)	(0.0127)	(0.0090)
Grils	-0.0850***	-0.1626***	-0.1711***	-0.1771***	-0.1511***	-0.0726***
	(0.0117)	(0.0141)	(0.0151)	(0.0155)	(0.0120)	(0.0083)

Table 8: Marginal Effect of PVE factors

	Low Cognitive			Medium Cognitive			High Cognitive		
Non-Cog:	Low	Med	High	Low	Med	High	Low	Med	High
Marginal Effect of High to Low									
Boys	-0.0500***	-0.2023***	-0.3841***	-0.1950***	-0.3806***	-0.3494***	-0.3010***	-0.4002***	-0.2509***
	(0.0088)	(0.0219)	(0.0271)	(0.0168)	(0.0249)	(0.0244)	(0.0243)	(0.0263)	(0.0197)
Girls	-0.0892***	-0.2452***	-0.3865***	-0.2662***	-0.3934***	-0.3347***	-0.3583***	-0.3819***	-0.2362***
	(0.0130)	(0.0226)	(0.0233)	(0.0166)	(0.0224)	(0.0234)	(0.0215)	(0.0238)	(0.0185)

Table 9: Decomposition of Gender Gap in University Participation

		Male Parameters			Female Parameters		
		Explained by		**Unexplained**	**Explained by**		**Unexplained**
		Observed Char.	Parameter(s)		Observed Char.	Parameter(s)	
Row	**Predicted gap: .1496**						
(1)	Observed Characteristics	-0.0084		0.1580	-0.0098		0.1594
		(0.0029)		(0.0093)	(0.0031)		(0.0094)
(2)	Full factor structure		0.1323	0.0173		0.1329	0.0167
			(0.0513)	(0.0521)		(0.0513)	(0.0521)
	Both factor loading and distribution						
(3)	Cognitive		0.0613	0.0711		0.0583	0.0746
			(0.0177)	(0.0478)		(0.0177)	(0.0478)
(4)	Non-Cognitive		0.0310	0.1014		0.0297	0.1032
			(0.0361)	(0.0279)		(0.0361)	(0.0279)
(5)	VPE		0.0426	0.0897		0.0411	0.0918
			(0.0257)	(0.0427)		(0.0257)	(0.0427)
	Factor loading						
			(0.0456)	(0.0161)		(0.0456)	(0.0161)
(6)	Cognitive		0.0294	0.1029		0.0304	0.1025
			(0.0163)	(0.0489)		(0.0163)	(0.0489)
(7)	Non-cognitive		0.0002	0.1322		0.0001	0.1328
			(0.0341)	(0.0277)		(0.0341)	(0.0277)
(8)	VPE		0.0001	0.1323		0.0001	0.1328
			(0.0159)	(0.0453)		(0.0159)	(0.0453)
	Factor distribution						
			(0.0152)	(0.0491)		(0.0152)	(0.0491)
(9)	Cognitive		0.0278	0.1045		0.0313	0.1016
			(0.0075)	(0.0496)		(0.0075)	(0.0496)
(10)	Non-cognitive		0.0308	0.1015		0.0296	0.1033
			(0.0046)	(0.0535)		(0.0046)	(0.0535)
(11)	VPE		0.0426	0.0898		0.0410	0.0919
			(0.0139)	(0.0489)		(0.0139)	(0.0489)

Table 10: Correlation between PVE and PISA Math and Science Scores

	Math		**Science**	
	Males	**Females**	**Males**	**Females**
PISA reading scores	0.7552***	0.7512***	0.8589***	0.8599***
	(0.0127)	(0.0116)	(0.0093)	(0.0111)
Parental Valuation Factor	-1.0434	-0.7624	-0.6213	-0.4005
	(1.1074)	(0.9955)	(0.8025)	(0.8465)
Sample Size	2464	3002	2506	2961

Table 11: Main reasons for Parental Aspirations

	< **University Aspirations**		**University Aspirations**	
	Boys	**Girls**	**Boys**	**Girls**
Better job opportunities or pay	0.6111	0.5707	0.5325	0.4790
	(0.0122)	(0.0131)	(0.0090)	(0.0082)
Valuable for personal growth and learning	0.0970	0.1165	0.1514	0.1769
	(0.0087)	(0.0093)	(0.0064)	(0.0058)
Child's choice	0.1048	0.1166	0.0612	0.0943
	(0.0070)	(0.0075)	(0.0052)	(0.0047)
Best match with child's ability	0.0776	0.0752	0.1207	0.1107
	(0.0075)	(0.0080)	(0.0055)	(0.0050)
Other	0.1094	0.1211	0.1341	0.1391
	(0.0083)	(0.0088)	(0.0061)	(0.0055)

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A Predicted probabilities for all factor points of support

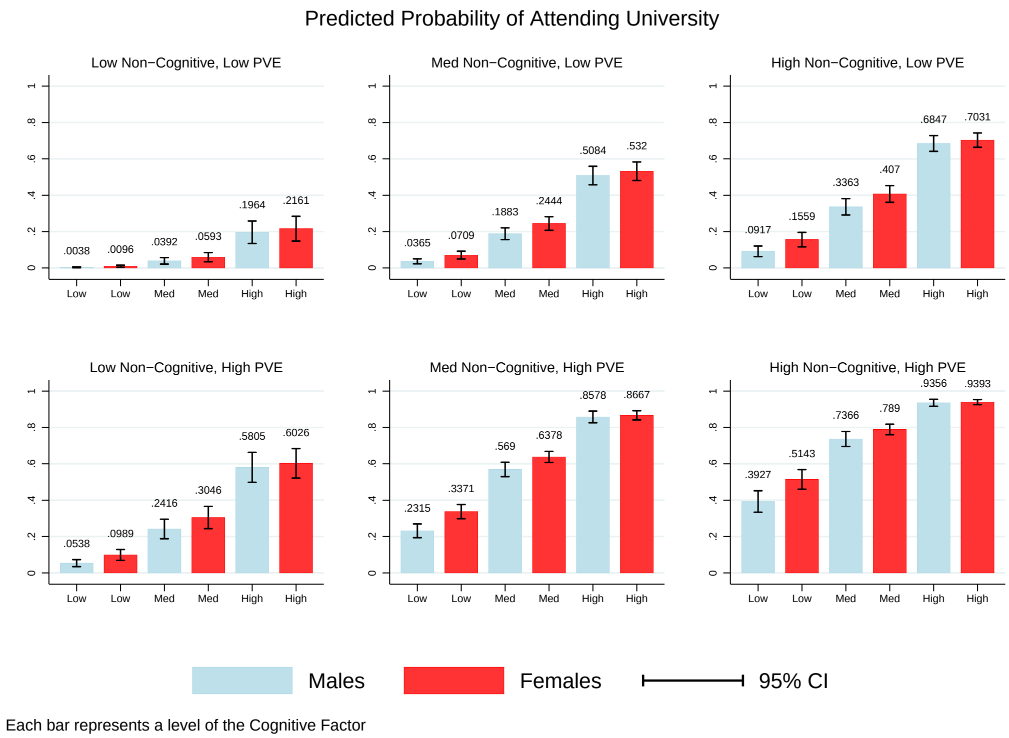

Figure 3: Predicted Probability of Attending University evaluated at each level of the estimated factors.

Notes: Confidence intervals constructed with standard errors estimated using the Delta Method and numerical derivatives.

B Detailed Decompositions

This appendix describes the detailed decompositions which are reported in columns 2 through 11 in Table 9

The predicted probabilities for males and females are, respectively:

$$U\left(X^m, m\right) = \frac{1}{n^m}\sum_{i=1}^{n^m}\sum_{\theta_1}\sum_{\theta_2}\sum_{v_p} p^m\left(\theta_1\right)p^m\left(\theta_2\right)p^m\left(v_p\right)F\left(\gamma_0^m + \gamma_x X_i^m + \lambda_{0\theta_1}^m\theta_{i1} + \lambda_{0\theta_2}^m\theta_{i2} + \lambda_{0v_p}^m v_{ip}\right)$$

$$U\left(X^f, f\right) = \frac{1}{n^f}\sum_{i=1}^{n^f}\sum_{\theta_1}\sum_{\theta_2}\sum_{v_p} p^f\left(\theta_1\right)p^f\left(\theta_2\right)p^f\left(v_p\right)F\left(\gamma_0^f + \gamma_x X_i^f + \lambda_{0\theta_1}^f\theta_{i1} + \lambda_{0\theta_2}^f\theta_{i2} + \lambda_{0v_p}^f v_{ip}\right)$$

The counterfactual for decomposing the fraction of the gap attributable to the entire factor structure is equation (13):

$$U\left(X^g, \gamma_0^g, h\right) = \frac{1}{n^g}\sum_{i=1}^{n^g}\sum_{\theta_1}\sum_{\theta_2}\sum_{v_p} p^h\left(\theta_1\right)p^h\left(\theta_2\right)p^h\left(v_p\right)F\left(\gamma_0^g + \gamma_x X_i^g + \lambda_{0\theta_1}^h\theta_{i1} + \lambda_{0\theta_2}^h\theta_{i2} + \lambda_{0v_p}^h v_{ip}\right)$$

The decomposition in Row 2 is:

Using the male parameters:

$$\Delta_{\Lambda\Theta^m} = \underbrace{U\left(X^f, f\right) - U\left(X^f, \gamma_0^f, m\right)}_{\text{Explained by factor structure}} + \underbrace{U\left(X^f, \gamma_0^f, m\right) - U\left(X^m, m\right)}_{\text{Unexplained}}$$

Using the female parameters:

$$\Delta_{\Lambda\Theta^f} = \underbrace{U\left(X^m, \gamma_0^m, f\right) - U\left(X^m, m\right)}_{\text{Explained by factor structure}} + \underbrace{U\left(X^f, f\right) - U\left(X^m, \gamma_0^m, f\right)}_{\text{Unexplained}}$$

The counterfactual in Row 3 is:

$$U\left(g, p^h\left(\theta_1\right), \lambda_{0\theta_1}^h\right) = \frac{1}{n^g}\sum_{i=1}^{n^g}\sum_{\theta_1}\sum_{\theta_2}\sum_{v_p} p^h\left(\theta_1\right)p^g\left(\theta_2\right)p^g\left(v_p\right)F\left(\gamma_0^g + \gamma_x X_i^g + \lambda_{0\theta_1}^h\theta_{i1} + \lambda_{0\theta_2}^g\theta_{i2} + \lambda_{0v_p}^g v_{ip}\right)$$

The decomposition in Row 3 is:

Using the male parameters:

$$U\left(X^f, f\right) - U\left(X^f, \gamma_0^f, m\right) = \underbrace{U\left(X^f, f\right) - U\left(f, p^m\left(\theta_1\right), \lambda_{0\theta_1}^m\right)}_{\text{Explained by } p^m\left(\theta_1\right) \text{ and } \lambda_{0\theta_1}^m} + \underbrace{U\left(f, p^m\left(\theta_1\right), \lambda_{0\theta_1}^m\right) - U\left(X^f, \gamma_0^f, m\right)}_{\text{Unexplained}}$$

Using the female parameters:

$$U\left(X^m, \gamma_0^m, f\right) - U\left(X^m, m\right) = \underbrace{U\left(m, p^f\left(\theta_1\right), \lambda_{0\theta_1}^f\right) - U\left(X^m, m\right)}_{\text{Explained by } p^f\left(\theta_1\right) \text{ and } \lambda_{0\theta_1}^f} + \underbrace{U\left(X^m, \gamma_0^m, f\right) - U\left(m, p^f\left(\theta_1\right), \lambda_{0\theta_1}^f\right)}_{\text{Unexplained}}$$

The counterfactual in Row 4 is:

$$U\left(g, p^h(\theta_2), \lambda^h_{0\theta_2}\right) = \frac{1}{n^g}\sum_{i=1}^{n^g}\sum_{\theta_1}\sum_{\theta_2}\sum_{v_p} p^g(\theta_1)\, p^h(\theta_2)\, p^g(v_p)\, F\left(\gamma_0^g + \gamma_x X_i^g + \lambda^g_{0\theta_1}\theta_{i1} + \lambda^h_{0\theta_2}\theta_{i2} + \lambda^g_{0v_p} v_{ip}\right)$$

The decomposition in Row 4 is:

Using the male parameters:

$$U\left(X^f, f\right) - U\left(X^f, \gamma_0^f, m\right) = \underbrace{U\left(X^f, f\right) - U\left(f, p^m(\theta_2), \lambda^m_{0\theta_2}\right)}_{\text{Explained by } p^m(\theta_2) \text{ and } \lambda^m_{0\theta_2}} + \underbrace{U\left(f, p^m(\theta_2), \lambda^m_{0\theta_2}\right) - U\left(X^f, \gamma_0^f, m\right)}_{\text{Unexplained}}$$

Using the female parameters:

$$U\left(X^m, \gamma_0^m, f\right) - U\left(X^m, m\right) = \underbrace{U\left(m, p^f(\theta_2), \lambda^f_{0\theta_2}\right) - U\left(X^m, m\right)}_{\text{Explained by } p^f(\theta_2) \text{ and } \lambda^f_{0\theta_2}} + \underbrace{U\left(X^m, \gamma_0^m, f\right) - U\left(m, p^f(\theta_2), \lambda^f_{0\theta_2}\right)}_{\text{Unexplained}}$$

The counterfactual in Row 5 is:

$$U\left(g, p^h(v_p), \lambda^h_{0v_p}\right) = \frac{1}{n^g}\sum_{i=1}^{n^g}\sum_{\theta_1}\sum_{\theta_2}\sum_{v_p} p^g(\theta_1)\, p^g(\theta_2)\, p^h(v_p)\, F\left(\gamma_0^g + \gamma_x X_i^g + \lambda^g_{0\theta_1}\theta_{i1} + \lambda^g_{0\theta_2}\theta_{i2} + \lambda^h_{0v_p} v_{ip}\right)$$

The decomposition in Row 5 is:

Using the male parameters:

$$U\left(X^f, f\right) - U\left(X^f, \gamma_0^f, m\right) = \underbrace{U\left(X^f, f\right) - U\left(f, p^m(v_p), \lambda^m_{0v_p}\right)}_{\text{Explained by } p^m(v_p) \text{ and } \lambda^m_{0v_p}} + \underbrace{U\left(f, p^m(v_p), \lambda^m_{0v_p}\right) - U\left(X^f, \gamma_0^f, m\right)}_{\text{Unexplained}}$$

Using the female parameters:

$$U\left(X^m, \gamma_0^m, f\right) - U\left(X^m, m\right) = \underbrace{U\left(m, p^f(v_p), \lambda^f_{0v_p}\right) - U\left(X^m, m\right)}_{\text{Explained by } p^f(v_p) \text{ and } \lambda^f_{0v_p}} + \underbrace{U\left(X^m, \gamma_0^m, f\right) - U\left(m, p^f(v_p), \lambda^f_{0v_p}\right)}_{\text{Unexplained}}$$

The counterfactual in Row 6 is:

$$U\left(g, \lambda^h_{0\theta_1}\right) = \frac{1}{n^g}\sum_{i=1}^{n^g}\sum_{\theta_1}\sum_{\theta_2}\sum_{v_p} p^h(\theta_1)\, p^g(\theta_2)\, p^g(v_p)\, F\left(\gamma_0^g + \gamma_x X_i^g + \lambda^h_{0\theta_1}\theta_{i1} + \lambda^g_{0\theta_2}\theta_{i2} + \lambda^g_{0v_p} v_{ip}\right)$$

The decomposition in Row 6 is:

Using the male parameters:

$$U\left(X^f,f\right)-U\left(X^f,\gamma_0^f,m\right)=\underbrace{U\left(X^f,f\right)-U\left(f,\lambda_{0\theta_1}^m\right)}_{\text{Explained by }\lambda_{0\theta_1}^m}+\underbrace{U\left(f,\lambda_{0\theta_1}^m\right)-U\left(X^f,\gamma_0^f,m\right)}_{\text{Unexplained}}$$

Using the female parameters:

$$U\left(X^m,\gamma_0^m,f\right)-U\left(X^m,m\right)=\underbrace{U\left(m,\lambda_{0\theta_1}^f\right)-U\left(X^m,m\right)}_{\text{Explained by }\lambda_{0\theta_1}^f}+\underbrace{U\left(X^m,\gamma_0^m,f\right)-U\left(m,\lambda_{0\theta_1}^f\right)}_{\text{Unexplained}}$$

The counterfactual in Row 7 is:

$$U\left(g,\lambda_{0\theta_2}^h\right)=\frac{1}{n^g}\sum_{i=1}^{n^g}\sum_{\theta_1}\sum_{\theta_2}\sum_{v_p}p^g\left(\theta_1\right)p^g\left(\theta_2\right)p^g\left(v_p\right)F\left(\gamma_0^g+\gamma_xX_i^g+\lambda_{0\theta_1}^g\theta_{i1}+\lambda_{0\theta_2}^h\theta_{i2}+\lambda_{0v_p}^gv_{ip}\right)$$

The decomposition in Row 7 is:

Using the male parameters:

$$U\left(X^f,f\right)-U\left(X^f,\gamma_0^f,m\right)=\underbrace{U\left(X^f,f\right)-U\left(f,\lambda_{0\theta_2}^m\right)}_{\text{Explained by }\lambda_{0\theta_2}^m}+\underbrace{U\left(f,\lambda_{0\theta_2}^m\right)-U\left(X^f,\gamma_0^f,m\right)}_{\text{Unexplained}}$$

Using the female parameters:

$$U\left(X^m,\gamma_0^m,f\right)-U\left(X^m,m\right)=\underbrace{U\left(m,\lambda_{0\theta_2}^f\right)-U\left(X^m,m\right)}_{\text{Explained by }\lambda_{0\theta_2}^f}+\underbrace{U\left(X^m,\gamma_0^m,f\right)-U\left(m,\lambda_{0\theta_2}^f\right)}_{\text{Unexplained}}$$

The counterfactual in Row 8 is:

$$U\left(g,\lambda_{0v_p}^h\right)=\frac{1}{n^g}\sum_{i=1}^{n^g}\sum_{\theta_1}\sum_{\theta_2}\sum_{v_p}p^g\left(\theta_1\right)p^g\left(\theta_2\right)p^g\left(v_p\right)F\left(\gamma_0^g+\gamma_xX_i^g+\lambda_{0\theta_1}^g\theta_{i1}+\lambda_{0\theta_2}^g\theta_{i2}+\lambda_{0v_p}^hv_{ip}\right)$$

The decomposition in Row 8 is:

Using the male parameters:

$$U\left(X^f,f\right)-U\left(X^f,\gamma_0^f,m\right)=\underbrace{U\left(X^f,f\right)-U\left(f,\lambda_{0v_p}^m\right)}_{\text{Explained by }\lambda_{0v_p}^m}+\underbrace{U\left(f,\lambda_{0v_p}^m\right)-U\left(X^f,\gamma_0^f,m\right)}_{\text{Unexplained}}$$

Using the female parameters:

$$U\left(X^m,\gamma_0^m,f\right)-U\left(X^m,m\right)=\underbrace{U\left(m,\lambda_{0v_p}^f\right)-U\left(X^m,m\right)}_{\text{Explained by }\lambda_{0v_p}^f}+\underbrace{U\left(X^m,\gamma_0^m,f\right)-U\left(m,\lambda_{0v_p}^f\right)}_{\text{Unexplained}}$$

The counterfactual in Row 9 is:

$$U\left(g,p^{h}\left(\theta_{1}\right)\right) = \frac{1}{n^{g}}\sum_{i=1}^{n^{g}}\sum_{\theta_{1}}\sum_{\theta_{2}}\sum_{v_{p}}p^{h}\left(\theta_{1}\right)p^{g}\left(\theta_{2}\right)p^{g}\left(v_{p}\right)F\left(\gamma_{0}^{g}+\gamma_{x}X_{i}^{g}+\lambda_{0\theta_{1}}^{g}\theta_{i1}+\lambda_{0\theta_{2}}^{g}\theta_{i2}+\lambda_{0v_{p}}^{g}v_{ip}\right)$$

The decomposition in Row 9 is:

Using the male parameters:

$$U\left(X^{f},f\right)-U\left(X^{f},\gamma_{0}^{f},m\right) = \underbrace{U\left(X^{f},f\right)-U\left(f,p^{m}\left(\theta_{1}\right)\right)}_{\text{Explained by } p^{m}\left(\theta_{1}\right)} + \underbrace{U\left(f,p^{m}\left(\theta_{1}\right)\right)-U\left(X^{f},\gamma_{0}^{f},m\right)}_{\text{Unexplained}}$$

Using the female parameters:

$$U\left(X^{m},\gamma_{0}^{m},f\right)-U\left(X^{m},m\right) = \underbrace{U\left(m,p^{f}\left(\theta_{1}\right)\right)-U\left(X^{m},m\right)}_{\text{Explained by } p^{f}\left(\theta_{1}\right)} + \underbrace{U\left(X^{m},\gamma_{0}^{m},f\right)-U\left(m,p^{f}\left(\theta_{1}\right)\right)}_{\text{Unexplained}}$$

The counterfactual in Row 10 is:

$$U\left(g,p^{h}\left(\theta_{2}\right)\right) = \frac{1}{n^{g}}\sum_{i=1}^{n^{g}}\sum_{\theta_{1}}\sum_{\theta_{2}}\sum_{v_{p}}p^{g}\left(\theta_{1}\right)p^{h}\left(\theta_{2}\right)p^{g}\left(v_{p}\right)F\left(\gamma_{0}^{g}+\gamma_{x}X_{i}^{g}+\lambda_{0\theta_{1}}^{g}\theta_{i1}+\lambda_{0\theta_{2}}^{g}\theta_{i2}+\lambda_{0v_{p}}^{g}v_{ip}\right)$$

The decomposition in Row 10 is:

Using the male parameters:

$$U\left(X^{f},f\right)-U\left(X^{f},\gamma_{0}^{f},m\right) = \underbrace{U\left(X^{f},f\right)-U\left(f,p^{m}\left(\theta_{2}\right)\right)}_{\text{Explained by } p^{m}\left(\theta_{2}\right)} + \underbrace{U\left(f,p^{m}\left(\theta_{2}\right)\right)-U\left(X^{f},\gamma_{0}^{f},m\right)}_{\text{Unexplained}}$$

Using the female parameters:

$$U\left(X^{m},\gamma_{0}^{m},f\right)-U\left(X^{m},m\right) = \underbrace{U\left(m,p^{f}\left(\theta_{2}\right)\right)-U\left(X^{m},m\right)}_{\text{Explained by } p^{f}\left(\theta_{2}\right)} + \underbrace{U\left(X^{m},\gamma_{0}^{m},f\right)-U\left(m,p^{f}\left(\theta_{2}\right)\right)}_{\text{Unexplained}}$$

The counterfactual in Row 11 is:

$$U\left(g,p^{h}\left(v_{p}\right)\right) = \frac{1}{n^{g}}\sum_{i=1}^{n^{g}}\sum_{\theta_{1}}\sum_{\theta_{2}}\sum_{v_{p}}p^{g}\left(\theta_{1}\right)p^{g}\left(\theta_{2}\right)p^{h}\left(v_{p}\right)F\left(\gamma_{0}^{g}+\gamma_{x}X_{i}^{g}+\lambda_{0\theta_{1}}^{g}\theta_{i1}+\lambda_{0\theta_{2}}^{g}\theta_{i2}+\lambda_{0v_{p}}^{g}v_{ip}\right)$$

The decomposition in Row 11 is:

Using the male parameters:

$$U\left(X^{f},f\right)-U\left(X^{f},\gamma_{0}^{f},m\right) = \underbrace{U\left(X^{f},f\right)-U\left(f,p^{m}\left(v_{p}\right)\right)}_{\text{Explained by } p^{m}\left(v_{p}\right)} + \underbrace{U\left(f,p^{m}\left(v_{p}\right)\right)-U\left(X^{f},\gamma_{0}^{f},m\right)}_{\text{Unexplained}}$$

Using the female parameters:

$$U\left(X^{m},\gamma_{0}^{m},f\right)-U\left(X^{m},m\right) = \underbrace{U\left(m,p^{f}\left(v_{p}\right)\right)-U\left(X^{m},m\right)}_{\text{Explained by } p^{f}\left(v_{p}\right)} + \underbrace{U\left(X^{m},\gamma_{0}^{m},f\right)-U\left(m,p^{f}\left(v_{p}\right)\right)}_{\text{Unexplained}}$$