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Adverse Selection in the Labour Market and the Demand for Vocational Education

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Adverse selection in the labour market and the demand for vocational education

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Abstract

In this paper, I show that displacement of high-school workers from routine jobs can be understood as the labour-market response to an adverse selection problem. The adverse selection problem arises because employment contracts do not systematically discriminate against education, even though over-qualified workers are relatively more likely to quit routine jobs. The labour market equilibrium distorts the labour market outcomes of high school graduates by inefficiently increasing their wage at the expense of higher unemployment rate, in order to separate them from overqualified college graduates. In addition, the labour market response to the adverse selection problem creates a demand for post-secondary vocational education, which is valuable because it acts as an entry barrier that prevents college graduates from using routine jobs as stepping-stones towards better jobs.

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1 Introduction

There is a common perception that high school graduates are being displaced from jobs involving mainly routine tasks. These jobs are increasingly automated, offshored, or performed by over-qualified workers, that is, workers whose qualifications exceed job requirements. The roles of skill-biased technological change and globalization in the determination of labour market conditions faced by less educated workers are widely recognized,[1] but the role of over-qualification remains unclear. Many studies assume that educated and uneducated workers compete for the same scarce jobs.[2] Yet, firm-level data does not support the view that over-qualified workers directly crowd out less educated workers.[3] Moreover, over-qualified workers are likely to use their current job as a stepping stone towards better jobs. In fact, more than 30 percent of overeducated workers switch into matched jobs within a year.[4] But if over-qualified workers have a relatively higher quit rate, it is unclear why they should be the ones displacing other workers, especially from routine jobs, where differences in productivity are not likely to be very large.

In this paper, I show that displacement of high-school graduates from employment can be understood as the labour-market response to an adverse selection problem. The adverse selection problem arises because employment contracts do not systematically discriminate against education, even though over-qualified workers are relatively more likely to quit those routine jobs. Comparing with high school graduates, overqualified college graduates are less attracted to routine jobs that offer a higher wage but is associated with a lower matching rate. Consequently, the labour market equilibrium generates inefficient unemployment of high school graduates. Moreover, I argue that this mechanism explains why vocational education has a higher market value than it is commonly thought.

Most studies of the role of education in the labour market focus on high school and college

[1] See Autor et al. (2003) on routine jobs, Goos et al. (2014) on skill biased technological change versus globalization and Autor et al. (2014) on exposure to international trade.

[2] See Gautier (2002), Albrecht and Vroman (2002), Dolado et al. (2009), Acemoglu and Autor (2011), Beaudry et al. (2013) and Barnichon and Zylberberg (2014).

[3] Gautier et al. (2002) find no evidence at the firm-level that firms upgrade their work force during low employment years.

[4] See Clark et al. (2014). Also see Van den Berg et al. (2002) for an example of stepping stone jobs.

workers. Yet, the increasing role of vocational education, which is the post-secondary education that focuses primarily on providing occupationally specific preparation, is striking. In 2010, for example, about 1,650,000 bachelor degrees and about 1,442,000 vocational education and training (VET) credentials were awarded in the U.S. VET credentials awarded per year increased by 60 percent from 2000 to 2010, growing much faster than the 34 percent increase of bachelor degrees. In this paper I argue that an important part of the labour-market value of vocational education stems from the fact that it acts as an entry barrier to exclude overqualified college graduates from some routine jobs. In this context, skill-biased technological change can explain the rise of vocational education over the past decades.

I begin by analyzing a dynamic frictional labour market with two types of workers and two types of jobs. I have in mind jobs that involve mainly cognitive tasks, but some of those jobs involve mainly routine tasks and some involve mainly non-routine tasks. Workers are assumed to have different educational backgrounds and are referred to as high school and college workers. For simplicity, I assume that all workers can perform routine tasks, and that high school and college workers are equally productive in these tasks. However, only college workers can perform non-routine tasks. College workers employed in routine jobs can search for non-routine jobs while on-the-job.

In this context, routine jobs have value for college workers as a stepping stone toward non-routine jobs. Employers offering these jobs understand that college workers are relatively less profitable than high school workers, because their quit rate is higher. However, employment contracts cannot discriminate effectively against education — either because such a form of discrimination is illegal or because college graduates cannot be compelled to disclose their college degree. This creates an adverse selection problem.

I show that any competitive search equilibrium of the model separates college and high school graduates searching for routine jobs into different markets.[5] The separation, however, is at the cost

[5] I extend the work of Guerrieri et al. (2010) on competitive search equilibrium with adverse selection to study a dynamic labour market equilibrium that includes on-the-job search.

of generating inefficient unemployment for high school graduates. This is because college workers are less willing to wait for a routine job than high school workers are. College workers search for routine jobs to transition into better jobs, so they are willing to accept a relatively lower wage as long as the job-finding rate is sufficiently high. In contrast, high school graduates are more likely to wait longer for a relatively higher wage, since they have a longer expected job tenure. In equilibrium, jobs that attract high school graduates offer a relatively higher wage,[6] and a corresponding inefficiently low job-arrival rate to discourage educated workers from applying. As a result, high school workers are displaced from employment by over-qualified college workers.

In this context, I argue that post-secondary vocational education has a higher market value than is commonly thought. The displacement effect through adverse selection is the key to understanding the value of vocational education as an entry barrier. Besides the obvious benefit of skill development, vocational education acts as an entry barrier because jobs requiring vocational education can exclude over-qualified workers from seeking stepping-stone opportunities. By extending the baseline model to include educational choices, I show that the adverse selection problem creates a demand for vocational education. Since employment contracts cannot discriminate against workers with more education, but can condition on required training, vocational education successfully excludes college workers from seeking stepping-stone opportunities. As a result, employers are able to offer contracts that do not suffer from the distortion of the adverse selection problem. I show that introducing costly vocational education into the labour market makes workers ex-ante better off. Vocational education helps screen workers who are more committed to the occupation and improves labour market efficiency.

Occupational licensing is an extreme example of entry barriers as it is often viewed to have no impact on productivity. According to Kleiner and Krueger (2013), in 2008 nearly 30 percent of the workers with more than high school education, but not a bachelor degree, were required to hold a license. Many popular fields of vocational education such as health care and "trades" in areas such as manufacturing, construction, repair, and transportation, also prepare students to

[6] Wage here represents the total labour income from the job that includes for example benefits and pensions. Jobs with high turnover rate are often less generous in these benefits.

obtain occupational licensing. One concern is that, as an entry barrier, occupational licensing may cause job losses by increasing employment costs. While this concern is widespread in policy papers (see Kleiner (2005)) and in the media, my results imply that occupational licensing also has an important benefit as an entry barrier. Imposing restrictions on entry provides protection for low to middle-skilled workers from the competition of college graduates seeking for stepping-stone jobs. Entry barriers such as vocational education and occupational licensing can mitigate the distortion caused by adverse selection by screening out workers who are not pursuing a career in the particular occupation. This is in sharp contrast to standard competitive market models, where a barrier to entry creates monopoly power and is unambiguously welfare reducing.

The adverse selection problem discussed in this paper and the role of vocational education as an enter barrier are similar to the standard signalling model of education (Spence, 1973). In the signalling model, a worker's unobserved productivity creates an adverse selection problem and educational credentials can be used to signal the worker's ability. In contrast, this paper highlights on-the-job search and the contractual incompleteness (through a firm's inability to exclude over-educated applicants) as a cause of a different adverse selection problem. Note that in this environment, workers with more education that are less desired by the routine employers because they are more likely to quit routine jobs in favour of non-routine jobs. Costly vocational education offers a way around the incomplete contract and acts as a screening device to exclude college workers from seeking stepping-stone opportunities.

Finally, I relate the outcomes of this model to the increase in post-secondary educational attainment over the past few decades. Autor et al. (2010) point out that one puzzle in the U.S. labour market is that the relative supply of college-educated workers is not growing fast enough, given the steep rise in the college-versus-high-school earnings ratio. This paper suggests that much of the increase in post-secondary education may take the form of non-bachelor post-secondary vocational education. The reason for this is that skill-biased technological change has not only increased the return to college, relative to high school, but also the return to vocational education. I simulate the model to examine the effect of a skill-biased technological change that is complementary to non-routine jobs but substitute to routine jobs, combined with an increase in university tuition fees to replicate the education of 2010. I find that while the increasing productivity of non-routine jobs leads to a direct increase in the educational attainment at university level, it also lowers the return to high school by further displacing high school workers from employment. Therefore such a change also increases the incentive for high school workers to attain both college and vocational education. In addition, the reduction in the productivity of some routine jobs has also worsened the labour market outcome of high school workers, leading to an increased popularity of VET. Overall, together with an increase in university tuition, skill-biased technological change can explain the increase of both college degrees and post-secondary VET certificates over the past thirty years.

The theory I propose is in contrast with previous work that emphasizes the displacement or "crowding-out" of less-educated workers by high skilled-workers. In the frictionless competitive models of Acemoglu and Autor (2011) and Beaudry et al. (2013), no externality is generated by employing high-skilled workers in less-skilled jobs, since replacing less-educated workers by high-skilled ones is an efficient market adjustment. In models with random matching, such as Gautier (2002), Albrecht and Vroman (2002) and Dolado et al. (2009), search externalities rely crucially on the assumption that high-skilled workers and less-educated workers are matched randomly with the same routine employers. What remains unexplained is whether this displacement exists when workers have incentives to apply to different employers. I show that over-qualified workers create a negative spillover effect on low-skilled workers even when they search for different routine jobs. Allowing workers to direct their search is crucial for understanding the mechanism that underlies the displacement of high school graduates and its policy implications.

The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 characterizes the equilibrium with the displacement effect. Section 4 extends the model with educational choices and discusses the implication on vocational education. I also present a numerical simulation to discuss the implications of skill biased technological changes. Section 5 concludes.

2 Model

2.1 Agents and Markets

Time is discrete and continues forever. All agents are risk neutral and discount the future at a rate $r > 0$. There are two types of workers, distinguished by their observable education attainment. Let $i \in \{1, 2\}$ indicate the types of workers, where type-2 workers can be interpreted as workers with a college/university degree while type-1 represents workers with a high school diploma only. The measure of workers is normalized to 1, with a fixed fraction π of college workers. There are also two types of jobs and $j \in \{1, 2\}$ indicate the job types. Type-2 jobs represent jobs that involve mostly non-routine cognitive tasks and require workers to have a college/bachelor degree. Type-1 jobs involve mostly routine tasks and the skill requirements are met by all workers. The measure of jobs is determined endogenously by employer's free entry. Each job requires only one worker to operate. Non-routine jobs produces output y_2 while routine jobs produces $y_1 < y_2$. Match productivity is summarized in Table 1. For simplicity I assume that college and high school workers are equally productive in preforming a routine job. The main results of the paper still hold if college workers are less or more productive than high school workers in a routine job.

Table 1: Match Productivity

Worker	Job	
	Non-college	College
High School	y_1	0
College	y_1	$y_2 > y_1$

There are four stages within each period: production, separation, search and matching. In the beginning of a period, all workers are either employed or unemployed. During the production stage, employed workers produce and collect wages while unemployed workers receive benefit b. During the separation stage, an exogenous separation shock occurs with probability $\delta > 0$. Once a match is hit by the shock, the worker becomes unemployed and starts job-searching in the next period while the job is destroyed. During the search stage, employed workers and workers who were unemployed at the beginning of the period can search. Meanwhile, firms create job vacancies and post wage contract for each vacancy. During the last stage, workers and employers are brought into contact to form new matches. When an employed worker receives an outside offer, her current employer can choose to make a take-it-or-leave-it counter-offer. The worker then chooses whether to accept the poaching offer and switch job or stay with the incumbent employer, in which case wages can only be renegotiated by mutual agreement.

To understand the impact of over-education on displacing high school workers, the equilibrium requires some fractions of college workers applying to non-college jobs. As the focus of this paper is about the labour market performance of high school workers rather than college workers, over-education is treated in a reduced-form way. I assume that when searching for college jobs, an unemployed college worker faces a cost $c > 0$. The cost is a random draw from a continuous distribution with cumulative distribution function $F(\cdot)$ and is i.i.d. across workers and over time. It is a relative cost and reflects the relative difficulty of applying for a college versus a non-college job for a college worker. The cost is revealed before the search stage and a college worker determines where to search based on the realization of the cost. For simplicity, I assume no cost is incurred when workers search on job, and none of my results rely on this assumption.

A worker's labour market state includes whether the worker is employed or unemployed, as well as the wage and the job type if the worker is employed. Let ℓ denote the labour market state of a worker. Given a wage ω and a job type j, the labour market state of an employed worker is a pair $\ell = (\omega, j)$. For an unemployed worker, her the labour market state is denoted by $(b, 0)$. The (full)state of a worker also includes her type, which is the education attainment of the worker. The state of a worker is perfectly observable.

Since the state of a worker is observable, firms can post a menu of contracts that offer different wages conditional on the state of a worker. With free entry of firms, an equilibrium where firms posting menus of contracts is equivalent to the equilibrium where each firm posts a single conditional contract in separated markets. In an equilibrium with menus, all posted menus must generate zero profit for the firms with each contract on the menu. This is because if firms can make positive profits with some contracts conditional on certain states, there is a profitable deviation for other firms to post menus that attract only workers with these states. Since all firms make zero profits with all contracts on a equilibrium menu, one can think equivalently as a market where employers posts contracts instead of menus. Without loss of generality, I focus on a labour market where each employer post a single wage contract conditional on workers' states. The contracting environment is that employers commit to the posted wage, which remains constant until an outside offer is received by the employee. Once an outside offer is received, employers can respond by offering a counter wage. The renegotiation of the wage is based on mutual agreement.

The key assumption of the model is that employers with non-college job vacancies cannot post contracts that are conditional on the education attainment of a worker. If employers could make distinct offers to college and high school workers, given that college workers are more likely to quit a routine job for a non-routine job, they would pay a lower wage to college workers for the same job. Restricting employers from offering conditional wage for a routine job eliminates discrimination against workers with more education. In practise, paying equally productive workers different wages might subject to legal risks and job posts that exclude workers with more education from applying are rare. One reason that such exclusion is difficult to implement is because committing not to hire over-qualified worker ex-ante is not credible ex-post. With search friction and costs of maintaining a vacancy, employer might prefer to hire an overqualified worker once they meet instead of leaving the vacant job unfilled. Another reason is that if employers try to exclude over-qualified workers through restricting job posts, workers might respond by omitting credentials that are not related to the job. Because employers cannot make contracts conditional on education for a routine job, they might face both college and non-college workers applying for the same vacancy. This particular contractual limitation causes an adverse selection problem that works as if education is not observable: being the less desired type for routine job employers, college workers might mimic the search behaviour of high school workers when applying for a routine job.

The structure of the labour market is as follows. Each period a continuum of markets may open. A market is characterized by an employment contract x and a market queue length q, which equal the measure of job applicants divided by the measure of vacancies in that particular market. Markets are indexed by contracts. A contract $x = \{(\omega, j), \ell\} \in X$ includes a job offer of a pair (ω, j) which specifies the wage ω and the job type j on the employer, as well as the labour market state of workers ℓ that the offer is made to. Notice that the contract is not conditional on the type of a worker because only college workers can preform a non-routine job and routine job employers are facing non-discrimination constraint. Let $Q : X \to \mathbb{R}^+$ be the queue mapping function and $Q(x)$ is the queue length associated with with a contract x. Search is directed as opposed to random matching. Each employer can post any feasible contract with a flow cost $k_j > 0$ for job type j and workers observe all posted wage and decide where to search. Both employers and workers take into account the trade-off that a wage is associate with higher market queue length for any given job.

Workers and firms in the same market are brought into contact by a matching technology in the matching stage. Matching is bilateral, so that each worker meets at most one employer and vice versa. Workers who search in a market with market queue length q meet an employer with probability $f(q)$, and employers in the same market meet a worker with probability $qf(q)$. Following the standard search literature, I assume that $f(q)$ is twice differentiable, strictly decreasing and convex, with $f(0) = 1$ and $f(\infty) = 0$. I also assume that $qf(q)$ is strictly increasing and concave, approaching 1 as q converges to ∞. In addition, I assume that the elasticity of the matching function with respect to job creation $\eta(q)$, defined as $\frac{qf'(q)}{f(q)}$ is concave, with $\eta(\infty) = 1$. This assumption is not necessary but simplifies the existence proof of an equilibrium.

I assume that $(r+\delta)k_2 < y_2 - y_1$ for all time in this paper. This assumption guarantees enough profits for employers to poach an employed worker and ensures that job-to-job transition is possible. On-the-job search plays an important role because the difference between a high school worker and a college worker, once employed in a routine job, are their on-the-job search opportunities. Without on-the-job search, college workers would search differently from high school workers for a routine job given their different reservation value of unemployment.

2.2 Worker's Problem

Let $s = \{\ell, i\} \in S$ denote the state of a worker of type i with labour force status ℓ and S be the set of all possible states of a worker. Let $V(s)$ denote the discounted lifetime income of a worker in state s. Notice that employed and unemployed workers search in different markets as specified by the required labour market states of a contract.

When unemployed, a worker receives benefit b and decides where to search after the search cost is realized. When an unemployed worker i searches for a job offer (ω, j) in market $x = \{(\omega, j), (b, 0)\}$, the probability of meeting an employer is $f(Q(x))$ and the state of the worker becomes $s_o = \{(\omega, j), i\}$ once matched. The value function $V(s)$ for a unemployed worker in state $s = \{(b, 0), i\}$ satisfies:

$$V(s) = b + \mathbb{E}_c \max_{(\omega,j)} \left\{ (1-i)(1-j)c + f(Q(x)) \max\left\{ \frac{V(s)}{1+r}, \frac{V(s_o)}{1+r} \right\} + [1 - f(Q(x))] \frac{V(s)}{1+r} \right\} \quad (1)$$

Similarly, employed workers collect wages and decides where to search on-the-job if survives the exogenous separation shock. Consider a type i worker who is currently employed in a type j job and receives wage ω. Given the worker's current labour market state $\ell = (\omega, j)$, if she searches for an offer (ω', j') in market $x = \{(\omega', j'), \ell\}$, the probability that she meets a poaching employer is $f(Q(x))$. Once the worker receives an offer (ω', j'), her current employer can respond by offering a counter-wage ω_c, which equals to zero when no counter-offer is made. If the worker accepts the outside offer, she switches to the new employer j' and forms a new match starting the next period with state $s_o = \{(\omega', j'), i\}$. Otherwise, the worker remains matched with her current employer. Her state becomes $s_c = \{(\omega_c, j'), i\}$ with new wage ω_c if the counter-offer is accepted, otherwise the worker remains in state $s = \{\ell, i\}$. The value function $V(s)$ of an employed worker in states $s \neq \{(b, 0), i\}$ satisfies:

$$V(s) = \omega + \frac{\delta V(s_u)}{1+r} + (1-\delta) \left\{ \max_{(\omega', j')} \left\{ f(Q(x)) \max\left\{ \frac{V(s)}{1+r}, \frac{V(s_o)}{1+r}, \frac{V(s_c)}{1+r} \right\} + [1 - f(Q(x))] \frac{V(s)}{1+r} \right\} \right\} \quad (2)$$

Denote by $g(s, c)$ the search policy of a worker in state s with cost realization c ($c = 0$ if the worker is employed). Also denote by $g^a(s, s_o, s_c)$ the acceptance policy such that $g^a(s, s_o, s_c)$ is the probability that a worker in state s accepts the outside offer. In equilibrium, employers take the worker's optimal search and acceptance policies as given.

2.3 Employer's Problem

Consider a type-j employer matched with a worker of type i receiving wage ω. Denote $H(s)$ the present value of an on-going match to the employer, where $s = \{(\omega, j), i\}$ is the state of the employee. $H(s)$ equals the discounted sum of profits for the employer matched with the worker in state s. Employers takes as given that if the match is not hit by the exogenous shock, the worker searches on-the-job according to the search policy g: the worker searches for outside offer $g(s, 0)$ in market $x' = \{g(s, 0), s\}$ and receives an outside offer with probability $f(Q(x'))$. Once an outside offer is received, the employer chooses whether to make a counter-offer ω_c, taking as given the the acceptance rule of the worker g^a. If the worker accepts the outside offer, her state becomes $s^o = \{g(s, 0), i\}$ and the current match is destroyed with zero value left for the employer. Otherwise, the worker stays with the incumbent employer. The state of the worker remains at s or changes to $s_c = \{(\omega_c, j), i\}$, if she accepts the counter offer. Since workers always apply for a higher wage when search on-the-job, optimal retention wage ω_c must be higher than the current wage. The present value of the match for the employer $H(s)$ satisfies the following:

$$H(s) = y_j - \omega + (1 - \delta) \left\{ f(Q(x')) \max_{\omega_c} \left\{ [1 - g^a(s, s_o, s_c)] \frac{H(s_c)}{1 + r} \right\} + [1 - f(Q(x'))] \frac{H(s)}{1 + r} \right\} \quad (3)$$

Denote the optimal retention policy of an employer by $g^r(s, s_o)$. The retention policy is contingent on the employee's current state.

Now consider firms with empty vacancies. Employers choose how many vacancies to create and where to locate them. Because contracts are conditional on workers' labour force status but not on their types, employers need to form expectations about the distribution of the applicants attracted to each posted contract. Let $\mu(\cdot|x)$ be the conditional distribution function of workers who are attracted by contract $x = \{(\omega, j), \ell\}$ across types. Let $J(x)$ be the ex-ante return of posting contract x to an employer. $J(x)$ is given by

$$J(x) = -k_j + Q(x) f(Q(x)) \sum_{i=1}^{2} \mu(i|x) g^a(s^i, s^i_o, s^i_c) \frac{H(s^i_o)}{1 + r} \quad (4)$$

where j is the job type posted. The states s^i, s^i_o, s^i_c of a worker of type i satisfy $s^i = \{\ell, i\}$,

$s_o^i = \{\{\omega, j\}, i\}$ and $s_c^i = \{g^r(s^i, s_o^i), i\}$ respectively for $i \in \{1, 2\}$.

2.4 Equilibrium Refinement

I now introduce a refinement that follows closely the equilibrium concepts in Chang (2012) and Guerrieri et al. (2010). Without any constraints on off-equilibrium beliefs, the model has numerous equilibria. The idea of the refinement is that a deviating contract must attract workers who have the strongest incentive to apply. Formally, denote $\tilde{V}(x, q, s)$ the expected return to a worker in state $s = \{\ell, i\}$ searching in market x with queue length q. The incentive of a worker i applying to deviating contract x is determined by the longest queue that a worker is willing to accept:

$$q(x, i) \equiv \sup\{\tilde{q} \geq 0 : \tilde{V}(x, \tilde{q}, s) \geq V(s)\} \tag{5}$$

Note that $\tilde{q} = 0$ if $\tilde{V}(\hat{x}, \tilde{q}, s) \geq V(s)$ has no solution. For contract x, $q(x, i)$ is the longest queue that a worker i is willing to accept in order to deviate. If $q(x, i)$ is positive, then the workers is indifferent between her equilibrium allocation and applying to contract x with queue length equal to $q(x, i)$. If x could attract some positive measure of workers, the type of the worker who has the strongest incentive to apply–the type with the longest $q(x, i)$–must have been attracted. Then the expectation function $\mu(\cdot|x)$ must satisfy that only workers who have the strongest incentive to apply are attracted. That is, belief must satisfy

$$\mu(i|x) = 0 \text{ if } i \notin \arg\sup_{i \in I}\{q(x, i)\} \text{ for any } x \in X. \tag{6}$$

This refinement arises naturally as a single employer does not serve the entire market under bilateral matching. It implies that when more that one type of worker is attracted by an off-equilibrium contract, their longest acceptable queue must be identical. It also implies that the queue mapping must satisfy:

$$Q(x) \equiv \sup_{i \in I} q(x, i)$$

The queue length associated with off-equilibrium contracts is determined by the longest acceptable queue of the attracted worker type. To understand this, consider the following adjustment process: if an employer posts an off-path contract x, some workers might find it profitable to apply if the market queue is sufficiently small. As workers flow in, the worker-over-employer ratio, i.e. the queue length increases. The flow-in of type i workers stops when they are indifferent between their equilibrium market and market x. They prefer not to apply to the market if the worker-over-employer ratio continues to increase if other types of workers with stronger incentives still flow in. This process stops when the queue length of the market reaches the longest acceptable queue of the worker type with the strongest incentive to apply. Therefore, the longest accepted queue of workers attracted to the contract determines $Q(x)$ for off-equilibrium contracts.

2.5 Equilibrium

Definition 1 *A stationary competitive search equilibrium consists of a set of posted contracts $X^* \subseteq X$, a market queue mapping Q, a value function for workers V, a value function for employers H, a search policy g, an acceptance rule g^a, a retention policy g^r, a conditional distribution function μ :, a distribution ϕ such that:*

(i) Workers optimize: $V(s)$ satisfies (1) when $s \in S^u$ and satisfies (2) for $s \in S \setminus S^u$ with $g(s,c)$ is the associated policy function and $g(s,c) \in X^*$ for all $c \in \mathbb{R}$, $s \in S$; and $g^a(s, s_o, s_c)$ is the acceptance rule such that

$$g^a(s, s_o, s_c) = \arg\max_{a \in [0,1]} \{aV(s_o) + (1-a)\max\{V(s), V(s_c)\}\} \tag{7}$$

(ii) Employers optimize: $H(s)$ satisfies (3) for all $s \in S^e$ with $g^r(s, s_o)$ the corresponding policy function;

(iii) For any $x \in X^*$, firms make zero profits: $J(x) = 0$

(iv) Beliefs are consistent: for any $x \in X^*$ and any state $\{\ell, i\} \in S$,

$$\mu(i|x) = \frac{\phi(\{\ell, i\})}{\sum_{i \in \{1,2\}} \phi(\{\ell, i\})}$$

(v) Steady-State conditions are satisfied.

(vi) Beliefs satisfy equilibrium refinement: $\mu(i|x)$ satisfies (6)

The general statement of the steady-state conditions is cumbersome and therefore not listed here, I provide the steady-state condition in the context of the equilibrium characterized below.

Condition (i) ensures that workers' search and acceptance policies are optimal for all states, taking as given the market queue length of all contracts and employer's optimal counter-offer strategy. Condition (ii) and (iii) ensures that, given the optimal search strategy of the worker, retention policies are optimal, and equilibrium contracts generates zero profits for employers. Condition (iii) ensures that for equilibrium contracts, employers beliefs are consistent with the workers search strategies through Bayes rule. Condition (v) ensures the law of motion for the aggregate state of the economy is stationary. Finally, condition (vi) restricts the off-equilibrium beliefs such that for any contract, only workers with the strongest incentives are attracted.

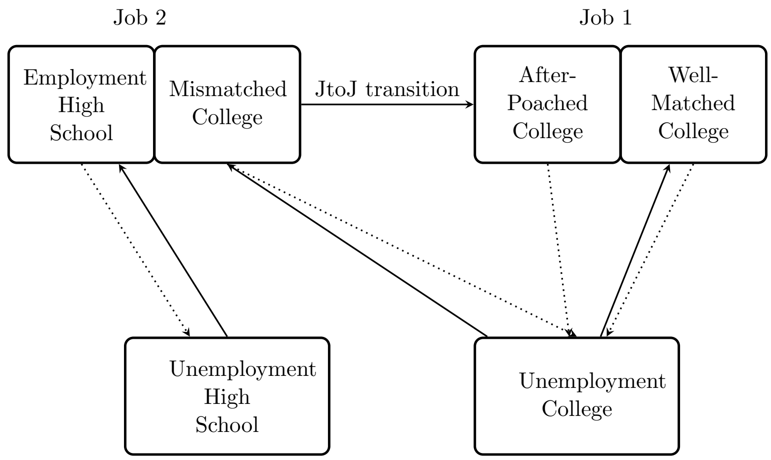
3 Equilibrium with Crowding-out

In this section, I study an equilibrium where the labour outcomes of high school workers are distorted with inefficiently high unemployment rate. The equilibrium can be understood by studying a set of constrained optimization problems. [7] The constrained optimization problems have an intuitive structure where workers maximize expected return to search subject to firms make non-negative profits and the incentive compatibility constraints of all types hold. After stating the problems, I show that a separating equilibrium can be supported with the solution of the problems and the equilibrium always exists. Under mild conditions, the incentive compatibility constraint of a college worker is binding and the equilibrium suffers from an adverse selection problem so that the unemployment rate of high school workers is inefficiently high.

Figure 1 presents the equilibrium flow of workers. High school workers search for non-college jobs only. Once employed, they do not search on-the-job. College workers can search for both college and non-college jobs when unemployed. An employed college worker search on-the-job only if when matched with a non-college job employer. Job-to-job transitions occur when a college worker switches from a non-college job employer to a college job employer. All employed workers face an

[7] See a full discussion in Guerrieri et al. (2010).

Figure 1: labour Market Flow



exogenous separation shock. Once hit by the shock, a worker becomes unemployed.

In equilibrium, job-to-job transitions occur only when switching employers generates a productivity gain. This is because the search cost is a sink cost for an employer who has already matched with a worker and it is optimal for the incumbent employer to counter offer up to the productivity of the match, once an outside offer is received by the employee. When there exists a negligible small cost of making offers, an employer find it not profitable to poach a worker who is equally productive with the current employer and with the poaching employer. Similarity, an incumbent employer do not make counter-offers if the workers is more productive with the poaching employer than in the current match. As a result, only college workers who are matched with non-college job employers search on-the-job. The search problems that need to be considered in the equilibrium are: the on-the-job search problem of an employed college worker, the search problem of an unemployed college worker for both college and non-college jobs and the search problem of an unemployed high school worker.

Consider first the on-the-job search problem of an employed college worker. Denote the value of a college worker who is currently employed in a non-college job with wage ω by $V(\{(\omega, 1), 2\})$. The worker searches for a college job and faces the following search problem. The worker chooses across all pairs of (ω', q) to maximize the expected return to search, subjects to the constraint that the poaching employer makes non-negative profits and the poaching wage is no less than her current matching productivity. For any given pair (ω', q), the worker meets an employer with probability $f(q)$ and forms a new match with value $V(\{(\omega', 2), 2\})$ to the worker with the new employer. With probability $1 - f(q)$, the worker stays with the incumbent employer and her state remains unchanged. $V(\{(\omega, 1), 2\})$ satisfies:

$$V(\{(\omega,1),2\}) = \omega + \max_{\omega', q}\left\{ f(q)\frac{V(\{(\omega',2),2\})}{1+r} + [1-f(q)]\frac{V(\{(\omega,1),2\})}{1+r} \right\} \qquad (\mathbf{P-1})$$

Subject to

$$-k_2 + qf(q)\left(\frac{y_2 - \omega'}{r+\delta}\right) \geq 0$$

$$\omega' \geq y_1$$

The value of a match with a college job for a college worker satisfies

$$\frac{V(\{(\omega,2),2\})}{1+r} = \frac{\omega}{r+\delta} + \frac{\delta}{r+\delta}\frac{V(\{(b,0),2\})}{(1+r)}$$

which equals to the discount sum of wages with the value of becoming unemployment $(V(\{(b,0),2\})$ with probability δ.

Denote the problem by $(\mathbf{P-1})$. The first constraint is the non-negative profits constraint for a college job employer searching for a worker in the market with wage offer ω' and queue length q. The employer pays cost k_2 to create the job and with probability $qf(q)$ she meets a worker. The expected profits of a match for the employers equals to $\frac{y_2 - \omega'}{r+\delta}$, which is the flow profits $y_2 - \omega'$ of the match discounted by the common discount rate r and the job separation risk δ. The second constraint restricts the equilibrium poaching wage to be no less than the worker's current productivity. Because the incumbent employer is able to make counter offers up to y_1, no poaching employer in equilibrium posts wages lower than y_1 to attract a mismatched college worker.

Denote the solution to the above problem by $\{q^e(\omega), \omega^e(\omega)\}$. Notice that the solution to the on-the-job problem depends on the worker's current wage. The probability that a college worker switches to a college job employer depends on the wage ω that she receives from the non-college job employer.

Next, I show the problem of an unemployed college worker searching for a non-college job employer, denoted by $(\mathbf{P-2})$. Denote the return to search for a non-college job to an unemployed college worker by V_2^1, such that

$$V_2^1 = \max_{\omega,q} \left\{ f(q)\frac{V(\{(\omega,1),2\})}{1+r} + [1-f(q)]\frac{V(\{(b,0),2\})}{1+r} \right\} \qquad (\mathbf{P-2})$$

s.t.

$$-k_1 + qf(q)\frac{y_1-\omega}{r+\delta+(1-\delta)f(q^e(\omega))} \leq 0$$

An unemployed college worker searches across all pairs of (ω, q) subject to the constraint that the non-college job employer makes non-negative profits. With probability $f(q)$, the worker meets an employer and is matched with a non-college job and starts search on-the-job in the next period. With probability $1-f(q)$, the worker does not find an employer and remain unemployed with value $V(\{(b,0),2\})$. Notice that the employer takes into consideration that the quit rate of the worker depends on the wage ω and discounts the expected profit of a match with $r+\delta+f(q^e(\omega))$, where $f(q^e(\omega))$ is the endogenous separation rate for the match.

Now consider the search problem of an unemployed college worker looking for a college job, denoted by $(\mathbf{P-3})$. As in the problem $(\mathbf{P-2})$, an unemployed college worker searches across all pairs of (ω, q) subject to the constraint that the college job employer makes non-negative profits. Different from searching for a non-college job, a college worker do not search on-the-job once matched with a college job. Denote the return to search for a college job to an unemployed college worker by V_2^2, such that

$$V_2^2 = \max_{(\omega,q)} \left\{ f(q)\frac{V(\{(\omega,2),2\})}{1+r} + [1-f(q)]\frac{V(\{(b,0),2\})}{1+r} \right\} \qquad (\mathbf{P-3})$$

s.t.

$$-k_2 + qf(q)\frac{y_2 - \omega}{r + \delta} \geq 0$$

The search problem is similar as the on-the-job search problem $(\mathbf{P-1})$, except that the workers remains unemployed with value $V(\{(b, 0), 2\})$ if she does not meet an employer within a period.

What remains in the search problems of a college worker, is to determine whether to search for a college job or a non-college job when unemployed. It is easy to see that there exists a cut-off level cost $\bar{c}$, such that a worker searches for a non-college job if and only if the realization of the search cost is larger than $\bar{c}$. Since the search cost c follows distribution with cumulative density function $F(c)$, an unemployed college worker searches en-ante for a college job with probability $F(\bar{c})$ with return to search V_2^2 and pays the cost. With probability $1 - F(\bar{c})$, the worker searches optimally for a non-college job with return to search V_2^1 and does not pay the search cost. The value of unemployment to a college worker $V(\{(b, 0), 2\})$ equals:

$$V(\{(b, 0), 2\}) = b + [1 - F(\bar{c})]V_2^1 + F(\bar{c})[V_2^2 - \mathbb{E}(c|c < \bar{c})]$$

The cut-off cost $\bar{c}$ must satisfy that an unemployed college worker with cost realization $\bar{c}$ is indifferent between searching for a college job and pay cost $\bar{c}$ or searching for a non-college job without paying the cost. The cut-off $\bar{c}$ is defined by the following equation:

$$V_2^2 - \bar{c} = V_2^1$$

Now consider the search problem of an unemployed high school worker. Like college workers, an unemployed high school worker maximizes the expected return to search by choosing across all pairs (ω, q) subjects to the constraint that the employer makes non-negative profits. Unlike college workers, the optimization problem of an unemployed high school workers is also subject to an incentive compatibility (IC) constraint. For any given pair (ω, q), the worker meets an employer with probability $f(q)$ and forms a match with value $V(\{(\omega, 1), 1\})$. Otherwise, the worker remain unemployed with the value denoted by $V(\{(b, 0), 1\})$. Denote the search problem of an unemployed high school worker by $(\mathbf{P-4})$, such that

$$V(\{(b,0),1\}) = b + \max_{\omega,q}\left\{f(q)\frac{V(\{(\omega,1),1\})}{1+r} + [1-f(q)]\frac{V(\{(b,0),1\})}{1+r}\right\} \qquad (\mathbf{P-4})$$

s.t.

$$-k_1 + qf(q)\frac{y_1-\omega}{r+\delta} \leq 0 \qquad \text{(Non-zero profits constraint)}$$

$$f(q)\frac{V(\{(\omega,1),2\})}{1+r} + [1-f(q)]\,\frac{V(\{(b,0),2\})}{1+r} \leq V_2^1 \qquad \text{(IC constraint)}$$

the value of a match for a high school worker satisfies:

$$\frac{V(\{(\omega,1),1\})}{1+r} = \frac{\omega}{r+\delta} + \frac{\delta}{r+\delta}\frac{V(\{(b,0),1\})}{(1+r)}$$

The IC constraint reflects the adverse selection problem and is the key to the displacement mechanism. Because of the non-discrimination against education restriction, contracts offered to high school workers can not exclude college workers from applying. At any given (ω, q), an employer expects less profits from matching with a college worker since the worker uses the job as a stepping-stone. The pair (ω, q) that makes an employer earning zero-profits by hiring a high school worker would generate negative profits if hiring a college worker. In equilibrium, no employer posts contracts to high school workers that would also attract college workers. The equilibrium contracts offered to high school workers must satisfy that a college worker weakly prefers search in her own market with value V_2^1 than searching for the same (ω, q) as high school workers, the value of which equals the left-hand-side of the IC constraint. One can see from problem $(\mathbf{P-1})$, when the IC constraint is binding, the solution to an unemployed high school worker's search problem is no longer constrained optimal.

I now show that the properties of a competitive search equilibrium can be understood by studying the above problems $(\mathbf{P-1})$ to $(\mathbf{P-4})$. Proposition 1 below establishes the linkage between the problems and the equilibrium.

Proposition 1 *i) Any equilibrium allocation must solve problems* $(\mathbf{P-1})$ *to* $(\mathbf{P-4})$*. ii)A solution to problems* $(\mathbf{P-1})$ *to* $(\mathbf{P-4})$ *can be supported as an equilibrium.*

The proof of this proposition follows closely after Guerrieri et al. (2010) and can be found in the appendix. Statement i) says that any solution to the set of problems generates an equilibrium. It is easy to see that the search problems in markets with college jobs are the standard competitive search problems without adverse selection. In markets with non-college jobs, however, the equilibrium expectation function is not degenerated because of the adverse selection problem. To show that workers maximize the expected return to search given the expectation function and the corresponding queue mapping function, I consider a truncated expectation function. For any wage post that is lower than the wage of an "indifference allocation", with which both the non-negative constraint and IC constraint in problem $(\mathbf{P-4})$ are binding, the employers expect only college workers to apply; otherwise the employers expect only high school workers to apply. With this expectation function and other corresponding equilibrium objects, the solution to problems $(\mathbf{P-1})$ to $(\mathbf{P-4})$ can be supported as a separating equilibrium, where college and high school workers search in different markets for a non-college employer.

The second statement of Proposition 1 says that one can find any equilibrium by solving problems $(\mathbf{P-1})$ to $(\mathbf{P-4})$. Notice that problems $(\mathbf{P-2})$ and $(\mathbf{P-4})$ are constructed under the assumption that an employer with a non-college job expects to meet either a college worker or a high school worker but not a mixed distribution of them. Therefore, statement ii) implies that any equilibrium must be separating. The explanation of no pooling equilibrium is as follows. If there were pooling in equilibrium, employers could attract the high school workers by offering a higher wage and break the proposed equilibrium. This is because unlike college workers who search for non-college jobs because they are a stepping stone to better jobs, high school workers are more willing to remain unemployed and search for jobs offering a relatively higher wage. The rational refinements on the off-path expectation function ensures that a deviating wage attracts the type of workers who have the strongest incentives to apply. Therefore, offering a higher wage attracts the desired type of workers, in which case are the high school workers, and is a profitable deviation.

Given that any solution to the set of problems ($\mathbf{P-1}$) to ($\mathbf{P-4}$) generates an equilibrium and any equilibrium must solve the problems, the existence of a separating equilibrium follows immediately.

Proposition 2 *There is a separating equilibrium.*

The sorting mechanism of the separating equilibrium is as follows. The market sorts unemployed workers by education according to their tolerance to remaining unemployed. College workers are less willing to remain unemployed, since they search for non-college jobs because they are a stepping stone to better jobs, so they are willing to search for jobs offering a relatively lower wage as long as the job finding rate is sufficiently high. In a separating equilibrium, the market makes the jobs that attract high school workers sufficiently hard to get so as to discourage educated workers from applying.

The next proposition states the main result of the paper. Under mild condition, the separating equilibrium suffers from an adverse selection problem and the labour market outcomes of high school workers are distorted.

Proposition 3 *There is a number $\bar{k} > 0$ such that for any $k_2 \in (0, \bar{k})$, an equilibrium suffer from adverse selection, and the unemployment rate of high school workers is inefficiently high.*

I show that when the cost of posting a college job is sufficiently small, the IC constraint of a college worker is binding. In this case, the constraint efficient allocation to a high school worker, which is the solution to the optimizing problem where high school workers maximizing expected return to search subject to employers making zero-profits, also attracts college workers. This means when the IC constraint is binding, no employer offers the wage that a high school worker finds optimal to apply. Instead, the market makes the jobs that attract high school workers sufficiently hard to get in order to exclude college workers from applying. The reason is that when searching for a non-college job, a college workers have lower tolerance to remaining unemployed than a high school worker. As a market solution to the adverse selection problem, the market sorts workers with different career paths by distorting the matching probabilities of high school workers.

In order for the market to distort the equilibrium matching rate and separate workers, the wage offered to high school workers is higher than its efficient level, even though the return to search, which is the discounted total labour income of a high school worker over time, is inefficiently low. In fact, the model implies that the skill premium, measured as the wage differential between high school and college workers, underestimates the true returns to education. This is because the true returns to education take into account not only the wage when a worker is employed, but also the job finding probability when a worker is unemployed, as well as the discounted value of future opportunities when a worker is employed. In this model, even though the skill premium is inefficiently low, as the wage of a high school workers is above its efficient level, the true returns to education are inefficiently high.

In the literature, many have discussed the crowding-out effect of over-qualified high-skilled workers on the employment of less-educated workers. These studies have taken as given that educated and uneducated workers do compete for the same scarce jobs. For example, in competitive framework with no search frictions, less-educated workers are replaced by more productive high-skilled workers in jobs that are less skill intensive and in routine tasks (see Acemoglu and Autor (2011) and Beaudry et al. (2013)). Yet firm level study has found no evidence supporting that firms upgrade their work force for low-skilled jobs in economic downturns (Gautier et al., 2002). In these models, no externality is generated by high-skilled workers employing in less-skilled jobs since replacing less-educated workers by high skilled ones is an efficient market adjustment. In search literature with random matching models, high-skilled workers create search externalities by applying to low-skilled jobs, which affects the creation of these jobs (see Gautier (2002), Albrecht and Vroman (2002) Dolado et al. (2009)). These externalities rely crucially on the assumption that high-skilled workers and less-educated workers are matched randomly with employers offering low-skilled jobs. However, the observation that the proportions of college workers vary across non-college occupations (Vedder, 2012) suggests that college workers do not search randomly for low-skilled jobs. Without imposing the assumption of direct competitions, the crowding-out effect of the above discussed models are not valid.

In this paper, the search behaviour of college workers generates a negative spillover effect on the unemployment rate of high school workers. The effect is similar to the crowding-out effect in the literature, but the mechanism is quite different. The high unemployment rate of high school workers is a market response to an adverse selection problem, where employment contracts cannot exclude college workers seeking non-college jobs as a stepping-stone. The displacement of high school workers is inefficient and is not caused by productivity driven adjustments. Even if college workers are less productive in non-college jobs, the distortions on high school workers still exists. The mechanism does not rely on the direct competition assumption between the workers neither. This means even without observing college and high school workers competing for the same jobs, the negative spillover effects caused by over-qualified college workers could still harm high school workers. Difference in mechanism presumably generates different implications. In the next section, I proceed by discussing the implication of this model on the post-second vocational education and training.

4 Vocational Education and Training

In the previous section, I show that the labour market responses to the adverse selection problem by distorting the labour market outcomes of high school workers. In the this section, I discuss the implication of the model. I argue that post-secondary vocational education and training (VET) has value as an entry barrier to college workers and is an institutional solution to the adverse selection problem.

VET here refers to the nonbaccalaureate post-secondary level education that focuses primarily on providing occupationally specific preparation. In the United State, VET has been growing fast in the past few decades. As showed in the beginning of the paper, the total number of VET certificates awarded per year has increased over 60 percent from 2000 to 2010, growing much faster than the 34 percent increase of bachelor degrees.[8] VET credentials take the form of either a post-secondary certificate or an associates degree and are provided mostly by community colleges in the U.S.

[8] National Center for Education Statistics (NCES).

In this section, I first argue that the fast-growing VET can be understood as a response to the adverse selection problem introduced in the previous sections. By acting as an entry barrier to college workers, VET helps improve the labour market outcomes of high school workers and increases labour market efficiency. I formalize this intuition by extending the baseline model to include educational choices. I show that VET is an useful labour market institution that mitigates the adverse selection problem. I also show that in contrast to the competitive models where barriers to entry are viewed as rent-seeking distortions to the market, introducing costly VET as an entry barrier to labour market is welfare improving. Finally, I preform a numerical simulation and examine the effects of skill-biased technological changes as well as an increase in college tuitions on the changes of educational choices.

4.1 VET as Entry Barrier for College Workers

With the insight of the mechanism presented in this paper, I argue that the displacement of high school workers caused by the adverse selection problem creates a demand for VET. The intuition behind such a demand is that VET has the value as an entry barrier to college workers. By requiring VET, a non-college job employer can exclude college workers from seeking stepping-stone opportunities and can therefore offer non-distorted wages to high school workers. By obtaining VET, high school workers gain access to markets that do not suffer from the adverse selection problem and improve their labour market performance. As long as the cost of VET does not excess the gain from receiving non-distorted labour market allocations, high school workers have incentives to pursue VET.

The adverse selection problem discussed in this paper and the role of VET as an enter barrier are similar to the standard Spence's signalling model of education (Spence, 1973). In the signalling model, worker's unobserved productivity creates an adverse selection problem and educational credentials can be used to signal worker's abilities. In this paper, the on-the-job search and the incompleteness of contracting to exclude over-educated applicants are the cause of the adverse selection problem. Notice that in this environment, workers with more education are the less desired type to the employers, because they are more likely to quit a lower-skilled job in favour of high-skilled jobs. In this sense, on-the-job search changes the direction of the adverse selection problem in terms of educational levels. Using VET, employers can condition job offers on required training and exclude college workers from seeking stepping-stones. Therefore, VET helps mitigate the adverse selection, like the role of educational credential in Spence's model, by resolving the contracting limitation of non-discrimination against education. Next, I formally present this intuition by extending the baseline model to include educational choices and VET.

4.2 A labour market with VET

Consider now a third type of jobs: the VET jobs (denoted by v). The difference between non-college jobs and VET jobs is that VET jobs require VET credentials while non-college jobs do not require any specific educational credential and can be performed by all workers. To separate the value of VET as an entry barrier from productivity, I assume in the model that the VET jobs and non-college jobs are equally productive such that $y_2 > y_1 = y_v$.

The distribution of workers' educational attainment is determined endogenously. All workers are ex-ante identical. They make a one-time educational choice at period zero before entering the labour market. The choice is made based on the cost and the expected return of education. Let e denote the cost of post-secondary education to a worker, where e follows a distribution with cumulative function $F_e(\cdot)$, and is i.i.d. across all workers. A worker chooses whether to pursue post-secondary education and within the post-secondary educational system whether to pursue a VET credential or a bachelor degree. A workers with cost e needs to pays αe for a bachelor degree and βe for VET with $\alpha > \beta$. In this model, all workers have high school diplomas.

The search problem of high school workers and college workers are identical as in the baseline model. A worker with a VET credential can apply for both non-college jobs and VET jobs. However, the worker in equilibrium applies to VET jobs only and do not search on-the-jobs once matched.[9] Unlike high school workers, however, the search problem for workers with VET is not subject to an adverse selection problem. Therefore workers with VET credentials receive efficient allocations by maximizing expected return to search subject to employers making zero-profits. Denote by U_h, U_v,

[9]For a worker with a VET credential, the cost of VET is a sunk cost and the optimal return to search for VET jobs weakly dominates the return to search for non-college jobs. Once matched with a VET job, there is no option value from on-the-job search, therefore the worker does not search on-the-job.

U_c the equilibrium return to search for a high school worker, a worker with a VET credential and a worker with a bachelor degree respectively. Within an educational group, all workers are identical.

To determine the distribution of workers, consider the following two cutoffs. Let $\bar{e}$ be the cutoff cost to post-secondary education so that a worker with educational cost $\bar{e}$ is indifferent between pursuing VET or obtaining a high school diploma only. The cutoff to post-secondary education $\bar{e}$ must satisfy

$$U_h = U_v - \beta\bar{e} \tag{8}$$

Similarly, let $\underline{e}$ be the cutoff so that a worker with educational cost $\underline{e}$ is indifferent between obtain a VET credential or a bachelor degree. The cutoff $\underline{e}$ must satisfy:

$$U_v - \beta\underline{e} = U_c - \alpha\underline{e} \tag{9}$$

If $\bar{e} \leq \underline{e}$, a worker chooses either to attain post-secondary education at university level or obtain only a high school diploma with no demand for VET. In the following proposition, I show that when the labour market suffers from the adverse selection problem and the cost of VET is not too large, the demand of VET is positive.

Proposition 4 *There is a level $\bar{\beta} \in (0, \alpha)$ such that for any $\beta \in (0, \bar{\beta})$, there exists an equilibrium with positive vocational education, for any $k_1 \in (0, \bar{k})$, with $\bar{k}$ defined in Proposition 3.*

When the cost of VET is not too large relative to the cost of university education, $\bar{e} > \underline{e}$. In this case, some workers find it optimal to pursue VET. In equilibrium, workers with an educational cost e smaller than $\underline{e}$ go to universities and can search for both non-college jobs and college jobs after graduation. Workers with a cost above $\bar{e}$ do not pursue post-secondary education and can only apply to non college jobs. Workers with a educational cost in between $\underline{e}$ and $\bar{e}$ choose VET. Workers with VET credentials can apply to both non-college and VET jobs, but would choose only VET in equilibrium. In this model, while the productivity gains of university education drives some workers to pursue post-secondary education, the entry barrier role of VET also encourages workers with only high school diploma to pursue post-secondary education. Next I discuss the welfare implications of VET.

At period 0, all workers are identical. Consider the situation when there is a demand for VET in the market. A worker makes educational choices based on the realization of her educational cost. With probability $F_e(\underline{e})$, the worker goes to universities for a college degree and pays the educational cost αe. With probability $1 - F_e(\bar{e})$, the worker holds a high school diploma only. With probability $F_e(\bar{e}) - F_e(\underline{e})$, the worker attains post-secondary education for VET and pays the educational cost βe. The ex-ante utility of a worker denoted by U_0 follows:

$$U_0 = F_e(\underline{e})U_c + [F_e(\bar{e}) - F_e(\underline{e})]U_v + [1 - F_e(\bar{e})]U_h - \beta\mathbb{E}[e|\underline{e} \leq e < \bar{e}] - \alpha\mathbb{E}[e|e \leq \underline{e}] \tag{10}$$

Since all employers in equilibrium make zero ex-ante profits, I can compare the total welfare of different economies by comparing the levels of U_0. Consider two labour markets with different educational institutions: one does not have VET and the other one offers VET which is costly. The follow proposition states the role of VET on improving welfare:

Proposition 5 *The optional to undertake VET makes workers better off ex-ante, for $k_1 \in (0, \bar{k})$ and $\beta \in (0, \bar{\beta})$.*

Introducing costly VET acting as an entry barrier for college workers can improve welfare by solving the adverse selection problem. VET attracts high school workers and workers who would choose university education when VET is absent. The gain from VET for high school workers is that workers with VET credentials gain access to jobs that exclude college workers and receive labour market outcomes that are not distorted by the adverse selection problem. Some workers who would pursue university education in the absence of VET also benefit from VET. These workers have relatively low educational costs amongst all workers so that they choose university level education. However, they have relatively high costs amongst college students so that their gain from the university education net the cost is small. With the option of VET, these workers benefit from applying to VET because even though the return to VET is less than the return to a university credential, the cost they could save from pursuing VET instead of university credential surpasses the difference in returns. So long as the cost to VET is not too large relative to gaining a bachelor degree, introducing VET to the labour market is strictly welfare improving.

One commonly debated form of entry barriers is occupational licensing. According to Kleiner and Krueger (2013), in 2008 nearly 30 percent of the workers with more than high school education, but not a bachelor degree, were required to hold a license. Many popular fields of VET such as health care and collective "trades" fields of manufacturing, construction, repair, and transportation, also prepare students for obtaining occupational licensing or certifications. One concern of the occupational licensing is that as an entry barrier, occupational licensing may cause job losses by increasing employment costs (see Kleiner (2005)). This is because in a market with perfect competitive, an entry barrier is a rent-seeking distortion to the market. However, using the insight of these paper, I argue that an entry barrier also has an important benefit to employment. In particular, imposing entry barriers on certain occupations provides protections for low to middle-skilled workers from the competition of over-qualified college workers seeking for stepping-stones.

Notice that the mechanism of the displacement of high school workers through adverse selection is the key to the welfare implication on VET. If displacement of high school workers happens in efficient labour market (see Acemoglu and Autor (2011), Beaudry et al. (2013)), a barrier to entry creates monopoly power and is a distortion to the market. Introducing new tasks or jobs that are equally productive as the existing ones but requiring additional training cost is not welfare improving. Even though low skilled workers could benefit from gaining access to jobs that exclude competition of college workers, the economy losses from incurring training cost without increasing productivity.

4.3 Post-Secondary Education and the Skill-Biased Technological Changes

In this section, I use some simulations of the model to help understand the effects of important labour market trends on the increase of post-secondary education in the past thirty years. In particular, I consider the effects of skill-biased technological changes (SBTC), which complement skilled cognitive jobs but substitute less-skilled routine jobs, [10] on educational choices. I also discuss

[10] See Autor et al. (2003) on routine jobs, Goos et al. (2014) on skill biased technological change versus globalization and Autor et al. (2014) on exposure to international trade.

the impact of an increase in college tuition on VET.

4.3.1 Benchmark Parameter Values

Assume the matching technology is giving by $M(u,v) = \dfrac{uv}{u+r}$. Condisder the following parameter values.[11] The productivity of non-college jobs y_1 is normalized to 1 and the productivity of college jobs y_2 equals 2.5. I relax the assumption that the productivity of VET jobs being equal to the productivity of non-college jobs and set $y_v = 1.5$. The flow income of unemployment b is set to be 0.1. Time is measured in quarters. Let $r = 0.01$, which is equivalent to an annual discount rate of 0.96. Given that job-to-job transition is endogenous in this model, the exogenous separation rate is set to be $\delta = 0.05$, consistent with the finding of Sahin et al. (2010). I assume the search cost c follows exponential distribution with parameter $\lambda = 1$ and the educational cost e follows beta distribution with parameter $(4, 4)$. The cost of posting college vacancies k equal to 0.2. The educational cost rate α and β are set to be 450 and 135 respectively, so that the educational attainment of the benchmark economy has 10 percent college graduates, 10 percent of VET workers and 80 percent high school workers, matching roughly the distribution of educational attainment in the 1970s. [12]

Column 2 in Table 2 presents the distribution of educational attainment and the key labour market outcomes of different workers in the benchmark economy. The economy has 80 percent high workers and 10 percent each for college and VET workers. Workers with more education have higher wage and lower unemployment rate. The college-versus-high-school wage ratio is 2.47, matching the college-versus-high-school earnings ratio in the early 80s (Acemoglu and Autor, 2011).

Next, I turn to simulations. I consider a positive shock to the productivity of college jobs, an negative shock to the productivity of non-college jobs and a scenario that combines both shocks with an increase in college tuition. The increase in productivity of VET jobs has straight forward effects and is not examined here.

[11] I found the IC constraint is always binding in all simulation examples with different parameters.

[12] U.S. Bureau of labour Statistics. I use here the group of workers with some college as well as workers with associate degrees to proximate VET workers.

Table 2

	Benchmark	Scenario 1	Scenario 2	Scenario 3
Education Attainment				
College	10%	98%	10%	33%
VET	10%	0	52%	30%
High School	80%	2%	38%	37%
Unemployment Rate				
College	6.0%	5.6%	6.0%	5.6%
VET	6.5%	6.5%	6.5%	6.5%
High School	9.6%	9.7%	10.3%	10.7%
Mean Wage				
College	2.42	4.92	2.42	4.92
VET	1.46	1.46	1.46	1.46
High School	0.98	0.98	0.68	0.68

4.3.2 Increased productivity of college jobs

Consider an increase in the productivity of college jobs with $y_2 = 5$. The results are presented in the second column of table 2 under Scenario 1. With a 100 percent increase in the productivity of college jobs while keeping the educational cost to university constant, 98 percent of workers would choose to become a college graduate and no worker prefers VET. The positive shock to college jobs has two effects on a worker's ex-ante educational choice. Increasing the productivity of college jobs raises the return to university education directly with a large increase in the wage and a mild decrease in the unemployment rate. It also lowers the return to search for a high school worker through the adverse selection effect. When ex-ante return to search for an unemployed college worker is higher, the worker becomes more "picky" about jobs even when applying for a stepping-stone job. As a result, the workers is more patient to remain unemployed and is less willing to accept a job with a lower wage, making it more difficult for the market to separate workers. The further distortion on high school workers are reflected in the increase of the unemployment rate and a negligible wage rise. The second effect, however, is quantitatively dominated by the first one.

4.3.3 Decreased productivity of non-college jobs

Now consider a fall on the productivity of non-college jobs by 30 percent so that $y_1 = 0.7$. This fall can be thought as automation and offshoring processes that substitute routine-based less-skilled workers. The results are presented in the third column of table 2 under Scenario 2. The negative shock reduces the return to search to an unemployed high school worker by decreasing the wage and increasing the unemployment rate. It increases the incentive for a high school graduate to obtain post-secondary education. However, the increase in post-secondary education is captured entirely by a 32 percent increase in VET without changing the fraction of university graduates. The negative shock on non-college jobs has little effect on the return to search for an unemployed college worker. This is because when a non-college jobs is less productive, fewer unemployed college workers choose to apply for a non-college job.

4.3.4 Increased university tuition

Finally, I consider a scenario with the two shocks discussed previously and a 100 percent increase in university cost α. As shown in the last column of Table 2 under Scenario 3, the economy has 33 percent college workers, 30 percent VET workers and 37 percent high school workers, similar to the education attainment in the U.S. in 2010.[13] The relative wage ratios between college workers versus VET workers and high school workers are 3.4 and 7.2 respectively. While the increased productivity on college jobs attracts workers to enrol in universities, the negative shock on non-college jobs and the increase in the cost to colleges relative to the cost to VET encourage workers to attain VET instead.

I relate these results to the argument of Autor et al. (2010) who points out that one puzzle in the U.S. labour market is that given the steep rise in the college-versus-high-school earnings ratio, the relative supply of college-educated workers is not growing fast enough. This paper shows that much of the increase in post-secondary education has taken the form of non-bachelor level post-secondary VET. Because skill-biased technological changes has increased not only the return to college relative to high school but also the return to non-university VET. Together with an increase in college tuition and costs, VET becomes an another popular educational investment amongst students.

[13]U.S. Bureau of labour Statistics.

5 Conclusion

In this paper, I argue that the labour market generates inefficient unemployment of high school graduates as a mechanism to separate high school graduates from overqualified college graduates searching for some types of routine jobs. This is how the labour market resolves the adverse selection problem arising from the fact that employment contracts in those routine jobs do not discriminate between high school and college graduates. In this context, the demand for vocational education arises because it allows employers to exclude applicants who treat the job as a stepping-stone. As a result, a vocational credential provides high school workers with access to markets that do not suffer from the distortion of over-educated workers and improve their labour market outcomes.

The model helps understand the effects of skill-biased technological change on the increase of post-secondary education in the past thirty years. Using numerical simulations, I show that skill biased technological changes increase the return to post-secondary education at both university level and non-university level in the form of vocational education. Together with an increase in college tuitions, skill biased technological changes can explain both the increase in college education and post-secondary vocational education over the past thirty years.

Appendix

Proof of Proposition 1

I first show that a solution exists to problem $(\mathbf{P-1})$ to $(\mathbf{P-4})$.

Lemma 1 *There exists a solution to* $(\mathbf{P-1})$ *to* $(\mathbf{P-4})$.

To show Lemma 1, consider first the problem $(\mathbf{P-3})$ of college workers. The first order conditions with respect to q and ω jointly gives

$$\frac{V(\{(\omega,2),2\})}{1+r}-\frac{V_2}{1+r}=\frac{1-\eta(q)}{\eta(q)}\frac{k_2}{qf(q)} \tag{11}$$

Substituting $\dfrac{V(\{(\omega,2),2\})}{1+r}$ and the zero-profit condition and denote the solution to this market (ω_2^2,q_2^2) which solves the following equations for any given V_2

$$\frac{y_2}{r+\delta}-\frac{r}{r+\delta}\frac{V_2}{1+r}=\frac{k_2}{\eta(q_2^2)q_2^2f(q_2^2)} \tag{12}$$

$$\omega_2^2=y_2-\frac{(r+\delta)k_2}{q_2^2f(q_2^2)} \tag{13}$$

The expected return to search for a college workers V_2^2 can be expressed as

$$V_2^2=\frac{1}{r}\left[y_2-\frac{k_2(r+\delta)}{\eta(q_2^2)q_2^2f(q_2^2)}\right]+\frac{1-\eta(q_2^2)}{\eta(q_2^2)}\frac{k_2}{q_2^2} \tag{14}$$

Next consider the on-the-job search problem $(\mathbf{P}-1)$. Ignore the last constraint, the first order condition for ω' and q jointly gives

$$\frac{\omega'}{r+\delta}+\frac{\delta}{r+\delta}\frac{V_2}{1+r}-\frac{V(\{(\omega,1),2\})}{1+r}=\left[\frac{1-\eta(q)}{\eta(q)}\right]\left(\frac{y_2-\omega'}{r+\delta}\right) \tag{15}$$

Substitute the stationary $V(\{(\omega,1),2\}$ from (28), an interior solution $q^e(\omega)$ and $\omega^e(\omega)$ satisfies

$$\frac{\omega^e(\omega)-\omega}{r+\delta+(1-\delta)f(q^e(\omega))}=\left[\frac{1-\eta(q)}{\eta(q)}\right]\left(\frac{y_2-\omega'}{r+\delta}\right) \tag{16}$$

together with the zero profit condition

$$\omega^e(\omega)=y_2-\frac{k_2(r+\delta)}{q^e(\omega)f(q^e(\omega))} \tag{17}$$

Denote q_a such that

$$q_af(q_a)\left(\frac{y_2-y_1}{r+\delta}\right)=k_2 \tag{18}$$

$q^e(\omega)\geq q_a$ if and only if $\omega^e(\omega)\geq y_1$. Also let q_b be such that

$$\frac{y_2 - y_1}{r+\delta} = \frac{k_2}{q_b f(q_b)} \left[1 + \frac{1-\eta(q_b)}{\eta(q_b)} \frac{r+\delta+(1-\delta)f(q_b)}{r+\delta}\right] \tag{19}$$

then $q^e(\omega) \le q_b$ if and only if $\omega \le y_1$. For any $\omega \in [0, y_1]$, the pair $(\omega^e(\omega), q^e(\omega))$ is uniquely given by (17) and (20) with $y_1 \le \omega^e(\omega) < y_2$ and $0 \le q_a \le q \le q_b < \infty$, where (20) is given by

$$\frac{\omega^e(\omega) - \omega}{r+\delta+(1-\delta)f(q^e(\omega))} \ge \left[\frac{1-\eta(q)}{\eta(q)}\right]\left(\frac{y_2 - \omega'}{r+\delta}\right) \tag{20}$$

with equality if $\omega^e(\omega) > y_1$. Rewrite equations (16) so that the wage from the off-job search problem is a function of the quit rate

$$\omega(q^e) = y_2 - \frac{k_2}{q^e f(q^e)} \left(r+\delta+\frac{1-\eta(q^e)}{\eta(q^e)}[r+\delta+(1-\delta)f(q^e)]\right) \tag{21}$$

Substituting ω and transform the surplus to the worker as a function of the quit rate. Denote $\hat{V}(q^e) \equiv \dfrac{V(\{(\omega,1),2\})}{1+r}$

$$\hat{V}(q^e) = \frac{y_2}{r+\delta} - \frac{k_2}{q^e f(q^e)\eta(q^e)} + \frac{\delta}{r+\delta}\frac{V_2}{1+r} \tag{22}$$

$$\frac{d\hat{V}(q^e)}{dq^e} = \frac{k_2}{q^e f(q^e)} \left(\frac{1}{q^e}\frac{1-\eta(q^e)}{\eta(q^e)} + \frac{\eta'(q^e)}{\eta^2(q^e)}\right) \tag{23}$$

Next consider the following transformed problem from $(\mathbf{P}-2)$. Using (22) the off-job search problem for a college worker looking for a non-college job can be reformed as

$$\max_{q',q} \left\{ f(q)\hat{V}(q') + [1-f(q)]\frac{V_2}{1+r} \right\} \tag{$\mathbf{P-2'}$}$$

s.t.

$$-k_1 + qf(q)\left[\hat{S}(q') - \hat{V}(q') - \frac{V_2}{1+r}\right] \le 0$$

Where $\hat{S}(q')$ is the total surplus of the match and $\hat{S}(q') - \hat{V}(q') - \dfrac{V_2}{1+r} = \dfrac{y_1 - \omega(q')}{r+\delta+(1-\delta)f(q')}$, with

$$\begin{aligned}\frac{d\left(\hat{S}(q^e) - d\hat{V}(q^e) + \frac{V_2}{1+r}\right)}{dq^e} &= \frac{(1-\delta)f'(q^e)}{[r+\delta+(1-\delta)f(q^e)]^2}\left(y_2 - y_1 - \frac{k_2(r+\delta)}{q^e f(q^e)}\right) \\ &- \frac{k_2}{q^e f(q^e)}\left[\frac{1-\eta(q^e)}{q^e}\left(\frac{1-\eta(q^e)}{\eta(q^e)} + \frac{r+\delta}{r+\delta+(1-\delta)f(q^e)}\right) + \frac{\eta'(q^e)}{[\eta(q^e)]^2}\right]\end{aligned} \tag{24}$$

I now solve the problem $(\mathbf{P}-\mathbf{2'})$ with the first order conditions with respect to q' and q given by

$$\lambda q = -\frac{\dfrac{d\hat{V}(q')}{dq'}}{\dfrac{d}{dq'}\left(\hat{S}(q') - \hat{V}(q') - \dfrac{V_2}{1+r}\right)} \tag{25}$$

$$\hat{V}(q') - \frac{V_2}{1+r} = \lambda q \left[\frac{1-\eta(q)}{\eta(q)}\right] \frac{k_1}{qf(q)} \tag{26}$$

Denote the solution to problem $\mathbf{P-2'}$ as $(q^e,\, q_2^1)$, which satisfies

$$\frac{y_2}{r+\delta} - \frac{k_2}{\eta(q^e)q^e f(q^e)} - \frac{r}{r+\delta}\frac{V_2}{1+r} = \lambda q_1^2 \left[\frac{1-\eta(q_1^2)}{\eta(q_1^2)}\right] \frac{k_1}{q_1^2 f(q_1^2)} \tag{27}$$

The return to search for a non-college job V_2^1 for college worker satisfies:

$$V_2^1 = \frac{1}{r}\left(y_2 - \frac{k_2(r+\delta)}{\eta(q^e)q^e f(q^e)} - \lambda q_2^1 \left[\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right] \frac{k_1(r+\delta)}{q_1^2 f(q_1^2)}\right) + \lambda q_2^1 \left[\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right] \frac{k_1}{q_2^1} \tag{28}$$

Substituting V_2^1 and V_2^2 from (28) and (14) into equation (3), the cut-off $\bar{c}$ solves

$$\bar{c} = \left[\frac{1-\eta(q_2^2)}{\eta(q_2^2)}\right] \frac{k_2}{q_2^2} - \lambda q_2^1 \left[\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right] \frac{k_1}{q_2^1} \tag{29}$$

From equation (3), the expected return to search for a college worker V_2 follows

$$V_2 - b = V_2^2 - \bar{c} + \nu(\bar{c}) = V_2^1 + \nu(\bar{c}) \tag{30}$$

where $\nu(\bar{c}) = F(\bar{c})[\bar{c} - \mathbb{E}(c|c \leq \bar{c})]$. The three equations listed below together equation (29) solves the search problem of a college worker with unknowns $\bar{c}$, q_2^1, q_2^2 and q^e.

$$\frac{y_2 - b + \bar{c} - \nu(\bar{c})}{r+\delta} = \frac{k_2}{\eta(q_2^2)q_2^2 f(q_2^2)} + \left[\frac{1-\eta(q_2^2)}{\eta(q_2^2)}\right] \frac{k_2}{(r+\delta)q_2^2} \tag{31}$$

$$\frac{y_2 - b - \nu(\bar{c})}{r+\delta} - \frac{k_2}{\eta(q^e)q^e f(q^e)} = \lambda q_2^1 \left[\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right] \frac{k_1}{f(q_2^1)q_2^1} + \lambda q_2^1 \left[\frac{1-\eta(q_2^1)}{\eta(q_2^1)}\right] \frac{k_1}{(r+\delta)q_2^1} \tag{32}$$

$$y_2 - \frac{k_2}{q^e f(q^e)}\left[r+\delta + \left[\frac{1-\eta(q^e)}{\eta(q^e)}\right](r+\delta+(1-\delta)f(q^e))\right] = y_1 - \frac{[r+\delta+(1-\delta)f(q^e)]k_1}{q_2^1 f(q_2^1)} \tag{33}$$

Next, I am going to show that there exists a solution for equations (31), (32) and (33) for a given $\bar{c} > 0$. I then follow by showing that a fix point exists for a function $D(\bar{c}) = \bar{c}) > 0$.

Fix a $\bar{c} > 0$. For equation (31), the right hand side converges to infinity when q_2^2 converges to 0 and converges to $k_2 + \dfrac{\nu(\bar{c}) - \bar{c}}{r+\delta}$. Therefore, there always exists a solution $q_2^2(\bar{c})$ such that equation (31) holds. For equation (32), since λq_2^1 is a function of q^e, notice first that $\dfrac{d(\lambda q_2^1)}{dq^e} < 0$. To see this, differentiate $\lambda q_2^1)$ with respect to q^e and notice that

$$\frac{\partial^2 \hat{S}/\partial(q^e)^2}{\partial^2 \hat{V}/\partial(q^e)^2} > \frac{\partial \hat{S}/\partial q^e}{\partial \hat{V}/\partial q^e}$$

The left side of the inequality is greater than one, since $\hat{S} - \hat{V}$ is a strictly convex function of q. The right side is smaller than one, since $\hat{S} - \hat{V}$ is a strictly decreasing function of q. Hence, $\frac{d(\lambda q_2^1)}{dq^e} < 0$, which suggests q_2^1 is a decreasing function of q^e in equation (32). The right hand side of equation (32) converges to infinity when q_2^1 converges to 0 and converges to $\frac{\nu(c)}{r+\delta}$ when q_2^1 converges to infinity. Therefore, there exists a $q_c(\bar{c}) > 0$ such that

$$\frac{y_2 - b}{r+\delta} - \frac{k_2}{\eta(q_c) q_c f(q_c)} = \frac{\nu(\bar{c})}{r+\delta}$$

and $q_c'(\bar{c}) > 0$. Then $q_2^1 < \infty$ if and only if $q^e > q_c(\bar{c})$. For equation (33), the left hand side converges to infinity when q_2^1 converges to 0 and to k_1 when q_2^1 converges to infinity. Define q_d such that

$$\frac{1}{r+\delta+(1-\delta)f(q_d)} \left[\frac{k_2(r+\delta)}{q_d f(q_d)} - (y_2 - y_1) \right] + \frac{k_2}{q_d f(q_d)} = k_1$$

and $q_2^1 < \infty$ if and only if $q^e < q_d > q_b$. Given that $q_a \leq q_b$, then for any $\bar{c}$, there is a solution to the equations (31), (32) and (33).

Now I construct a function $D(\bar{c})$ equal to the right hand side of (29). $D(0)$ satisfies

$$D(0) = \frac{k_1 \lambda q_2^1}{\eta(q_2^1) q_2^1 f(q_2^1)} - \frac{k_2}{\eta(q_2^2) q_2^2 f(q_2^2)} + \frac{y_2 - y_1}{r+\delta+(1-\delta)f(q^e)} + \frac{k_2(1-\delta)}{q^e} \frac{1}{r+\delta+(1-\delta)f(q^e)}$$

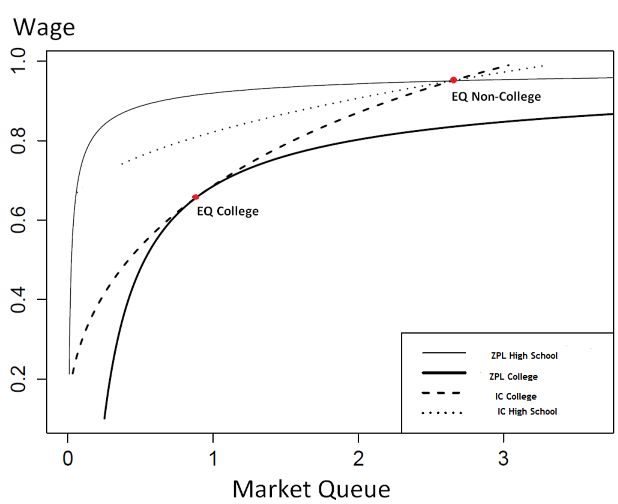
and $D(0) > 0$ with $q_2^2 > q_2^1$. From equation (31), when $\bar{c}$ increase, q_2^2 decrease, therefore the first item in $D(\bar{c})$ increases. From equation (32) and (33), when $\bar{c}$ increases, q_2^1 increases and q^e increases, therefore, the second term of $D(\bar{c})$ decreases and $D(\bar{c})$ is an increasing function of $\bar{c}$. Given that $D(0) > 0$ and $D(\bar{c})$ is finite, there exists a fixed point to $D(\bar{c}) - \bar{c}$ with $\bar{c} > 0$.

Given that a solution exists to the search problem of a college worker, the solution exist to the search problem of high school worker ($\mathbf{P} - 4$) is trivial. When the IC constraint is not binding, a high school worker faces the standard competitive search problem. When the IC constraint is binding, the equilibrium allocation of a high school worker is determined jointly by the zero-profit condition and a binding IC constraint stated in ($\mathbf{P} - 4$).

Now I prove the first statement of Proposition 1. I first state that any equilibrium must be separating. The reason why no pooling equilibrium is as follows. Consider a contract with wage ω in non-college job market such that both types of workers apply in equilibrium. Contract x with the corresponding queue q must lines between the zero-profit line of hiring high school workers alone and the zero-profit line of hiring college workers alone, in the space of wages and market queue as shown in Figure 2. However, at (q, ω), the slope of indifference curves of college workers is stepper than that of high school workers. This is because unlike college workers who search for non-college jobs because they are a stepping stone to better jobs, high school workers are more willing to remain unemployed and search for jobs offering a relatively higher wage. This means for a higher wage, high school workers are willing to accept a larger queue to be indifferent. Following the equilibrium refinement, there always exists a profitable deviation for a routine employer to post a wage higher than ω such that only high school workers are attracted. Therefore, pooling equilibrium does not exists.

Given that any equilibrium must be separating, the rest of the proof of statement 1 follows the equilibrium definition. By definition, an equilibrium must be that workers choose where to apply to maximize expected return to search and employer make zero profits, as is characterized in problems $(\mathbf{P}-1)$ to $(\mathbf{P}-4)$. Any equilibrium must be separating implies that the incentive compatibility constraint of a college worker must hold. This is guaranteed by the solution of problem $(\mathbf{P}-4)$. Therefore, any equilibrium allocation must solve problems $(\mathbf{P}-\mathbf{1})$ to $(\mathbf{P}-\mathbf{4})$.

Figure 2: Non-College Job Markets



Finally, I prove the second statement of Proposition 1 hold by constructing the construction. Denote the solution to problems $(\mathbf{P}-1)$ to $(\mathbf{P}-4)$ as the follow four pairs (ω_2^2, q_2^2), (ω^e, q^e), (ω_2^1, q_2^1),(ω_1, q_1). I first construct the objectives of a potential equilibrium and show that all the equilibrium conditions as defined in Definition 1 could be satisfied with these objectives.

- The set of contracts X^* includes: $\{(\omega_2^2, 2), (b, 0)\}$, $\{(\omega^e, 2), (\omega_2^1, 1)\}$, $\{(\omega_1, 1), (b, 0)\}$ and $\{(\omega_2^1, 1), (b, 0)\}$,

- The retention policy g^r such that for any $s = \{(\omega, j), i\}, s_o = \{(\omega', j'), i\}$:

$$g^r(s, s_o) = \begin{cases} 0, & \text{if } j < j' \\ \omega' & \text{if } j \geq j' \end{cases} \tag{34}$$

- The market queue mapping Q satisfies:

  i any wage ω posted from a college employer attracting unemployed college workers and employed college workers, Q is such

$$k = qf(q)\frac{y_2 - \omega}{r + \delta}$$

  ii any wage ω posted from a non-college job employer attracting unemployed workers Q is such that

$$\begin{cases} k_1 = qf(q)\dfrac{y_1 - \omega}{r + \delta + (1 - \delta)f(q^e(\omega))}, & \text{if } \omega < \omega_2^1 \\ V_2^1 = f(q)\frac{V(\{(\omega,1),2\})}{1+r} + [1 - f(q)]\dfrac{V_2}{1 + r}, & \text{if } \omega_2^1 \leq \omega \leq \omega_1 \\ k_1 = qf(q)\dfrac{y_1 - \omega}{r + \delta}, & \text{if } \omega > \omega_1 \end{cases} \tag{35}$$

- The conditional distribution function μ such

  i any contract x posted from an college job employer: $\mu[2|x] = 1$

  ii any contract $x = \{(\omega, 1), (b, 0)\}$ posted from an non-college job employer

$$\mu[2|x] = \begin{cases} 1, & \text{if } \omega < \omega_1 \\ 0, & \text{if } \omega \geq \omega_1 \end{cases} \tag{36}$$

- The distribution of workers $\phi = \{u_1, u_2, e_1, e_2^1, e_2^2(\omega_2^2), e_2^2(\omega^e)\}$ such that the inflow equals outflow at each state

$$\begin{aligned} &u_1 f(q_1) = (1 - \pi - u_1)\delta \\ &e_1 = 1 - \pi - u_1 \\ &u_2[F(\bar{c})f(q_2^2) + [1 - F(\bar{c})]f(q_2^1)] = (\pi - u_2)\delta \\ &e_2^1[\delta + (1 - \delta)f(q^e)] = u_2[1 - F(\bar{c})]f(q_2^1) \\ &e_2^1(1 - \delta)f(q^e) = \delta e_2^2(\omega^e) \\ &\pi - u_2 - e_2^1 - e_2^2(\omega^e) = e_2^2(\omega_2^2) \end{aligned}$$

I now show that all of equilibrium conditions from Definition 1 hold. Condition (i) In all college job market, since Q traces zero profit condition, workers maximize expected return to search given Q in $(\mathbf{P-1})$ and $(\mathbf{P-2})$. In non-college job market, Q ensures that college worker maximized given employer makes zero profits while high school worker maximized given the incentive compatibility constraint, as in $(\mathbf{P}-\mathbf{4})$. The search policy g is given by the solution of each optimization problem.

Condition (ii) The retention policy g^r maximize the continuation value of a match for employers and the zero-profit conditions satisfies (3). Condition (iii) has been shown in Lemma 1 that the zero profit condition binds for all equilibrium contracts. Condition (iv) and (v) are satisfied by the construction of μ and ϕ. Finally, the constructed expectation function μ also ensures that no pooling deviation is profitable because any deviation wage attracts at most one type of worker. *QED*

Proof of Proposition 2

Given than 1) a solution to problems $(\mathbf{P}-\mathbf{1})$ to $(\mathbf{P}-\mathbf{4})$; 2) any equilibrium must be separating and solve problems $(\mathbf{P}-\mathbf{1})$ to $(\mathbf{P}-\mathbf{4})$; 3) a solution to problems $(\mathbf{P}-\mathbf{1})$ to $(\mathbf{P}-\mathbf{4})$ can be supported as an equilibrium, there exists a separating equilibrium.

Proof of Proposition 3

I show that when the cost of posting college job k_2 converges to zero, the IC constraint is binding. When $k_2 \to 0$, $f(q^e) = 1$ and $\omega^e = y_2$. A non-college job employer receives one period profit from a college worker such. The optimal search problem for a college worker is :

$$V_2^1 = \max_{\omega, q} \left\{ f(q)\frac{V(\{(\omega,1),2\})}{1+r} + [1-f(q)]\frac{V(\{(b,0),2\})}{1+r} \right\} \tag{37}$$

s.t.

$$-k_1 + qf(q)\frac{y_1 - \omega}{1+r} \leq 0$$

where

$$V(\{(\omega,1),2\}) = \omega + \frac{\delta V(\{(b,0),2\})}{1+r} + \frac{(1-\delta)V(\{(y_2,2),2\})}{1+r}$$

The F.O.C. satisfies:

$$\frac{V(\{(\omega,1),2\})}{1+r} - \frac{V(\{(b,0),2\})}{1+r} = \frac{1-\eta(q)}{\eta(q)}\frac{k_1}{qf(q)} \tag{38}$$

Substituting $\dfrac{V(\{(\omega,1),2\})}{1+r}$ and the zero profit condition and denote q_0 the solution to the problem (37), then equation (38) becomes

$$\frac{y_1}{r+\delta} - \frac{(1-\delta)y_2}{(1+r)(r+\delta)} - \frac{rV(\{(b,0),2\})}{(1+r)(r+\delta)} = \frac{k_1}{\eta(q_0)q_0 f(q_0)} \tag{39}$$

Compare to the FOC of the high school worker's search problem where the IC constraint is absent, where q_1 denote the equilibrium queue length of a high school worker's allocation

$$\frac{y_1}{r+\delta} - \frac{rV(\{(b,0),1\})}{(1+r)(r+\delta)} = \frac{k_1}{\eta(q_1)q_1 f(q_1)} \tag{40}$$

Compare the two equations. Given than $V(\{(b,0),2\}) > V(\{(b,0),1\})$, and $\eta(q)qf(q)$ is an increasing function in q, $q_0 > q_1$. This implies that when k_2 converges to 0, a college worker strictly prefer the undistorted allocation of a high school worker with queue length q_1 to her own optimal allocation with queue length q_0.

To see this, consider the following three allocations: allocation (q_0, ω_0) is the optimal allocation of a college worker, (q_1, ω_1) is the undistorted optimal allocation of a high school worker as well as (q_0, ω_0') where ω_0' is the wage that generates zero-profits for an employer if the corresponding queue is q_0 and the worker does not search on-the-job. A college workers strictly prefer allocation (q_0, ω_0') to (q_0, ω_0) because for the same matching rate the wage ω_0' is larger than ω_0 when workers are expected to search on-the-job. A college workers also prefers allocation (q_1, ω_1) to (q_0, ω_0'). This is because at q_0, employers is welling to give up more wage for an increase in matching rate when they do not expect workers to search on the job.

Finally, since a college worker prefers (q_0, ω_0') to (q_0, ω_0), the equilibrium allocation (q_1^*, ω_1^*) of a high school worker that makes a college worker indifferent must have $q_1^* > q_0 > q_1$. The matching rate of a high school worker is lower than its efficient level and the unemployment rate of high school workers is inefficiently high.

Proof of Proposition 4

The cutoffs of post-secondary education $\bar{e} = \dfrac{U_v - U_h}{\beta}$ and the cutoff of pursuing bachelor degree $\underline{e} = \dfrac{U_c - U_v}{\alpha - \beta}$. Denote $\bar{\beta}$ such that $\bar{e} = \underline{e}$. At $\bar{\beta}$, a worker with cost $\bar{e}$ is indifferent between obtaining a bachelor degree or a VET credential. $\bar{\beta}$ solves:

$$\bar{\beta} = \alpha \frac{U_v - U_h}{U_c - U_h} \tag{41}$$

Given that $U_v > U_h$ and $U_c > U_h$, $\bar{\beta} > 0$ for any $\alpha > 0$. QED

Proof of Proposition 5

Fix α. Consider $\bar{e}(\beta) = \dfrac{U_v - U_h}{\beta}$ with $\dfrac{d\bar{e}}{d\beta} < 0$. Similarly, $\underline{e}(\beta) = \dfrac{U_c - U_v}{\alpha - \beta}$ with $\dfrac{d\bar{e}}{d\beta} > 0$. Consider the ex-ante utility of a worker U_0 as the function of β such that

$$U_0(\beta) = F_e(\underline{e}(\beta))U_c + [F_e(\bar{e}(\beta)) - F_e(\underline{e}(\beta))]U_v + [1 - F_e(\bar{e}(\beta))]U_h - \beta\mathbb{E}[e|\underline{e}(\beta) \le e < \bar{e}(\beta)] - \alpha\mathbb{E}[e|e \le \underline{e}(\beta)] \tag{42}$$

$$\frac{dU_0(\beta)}{d\beta} = -\int_{\underline{e}(\beta)}^{\bar{e}(\beta)} e f_e(e) de \tag{43}$$

The ex-ante utility of a worker in a economy without VET equals $U_0(\bar{\beta})$ with $\beta = \bar{\beta}$ defined in Proposition 4. At $\beta = \bar{\beta}$, $\frac{dU_0(\beta)}{d\beta} = 0$. The ex-ante utility of a worker in a economy with VET equals $U_0(\beta)$ with $\beta < \bar{\beta}$, in which case $\frac{dU_0(\beta)}{d\beta} < 0$ and the ex-ante utility of a worker is strictly increasing as β decreases. By continuity, $U_0(\bar{\beta}) < U_0(\beta)$ for any β such that $0 < \beta < \bar{\beta}$. The ex-ante utility of a worker is higher in economy with VET. Since all employers in equilibrium makes zero expected profits, the total ex-ante welfare of an economy increases by introducing VET. *QED*

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