Canadian Labour Economics Forum

WORKING PAPER SERIES

Labor Market Policies in a Roy-Rosen Bargaining Economy

Hugo Jales (University of Syracuse)
Zhengfei Yu (University of Tsukuba)

Spring-Summer 2020, WP#29

LABOR MARKET POLICIES IN A ROY-ROSEN BARGAINING ECONOMY

Hugo Jales [1] and Zhengfei Yu [2]

Abstract: We study the effects of labor market policies using a bargaining model featuring *compensating differentials* (Rosen, 1986) and *self-selection* (Roy, 1951). The framework allows us to create a taxonomy of formal and informal employment. We use the model to estimate the effects of the minimum wage for the Brazilian economy using the "PNAD" dataset for the years 2001-2005. Our results suggest that, although the minimum wage generates unemployment and reallocation of labor to the informal sector, the policy might be desirable if the employment losses are concentrated in jobs characterized by low surplus.

[1]Syracuse University, Department of economics. Email: hbjales@syr.edu.
[2]University of Tsukuba, Faculty of Humanities and Social Sciences.

1. INTRODUCTION

Labor markets in developing countries are usually characterized by a large number of informal employment relationships (Ulyssea, 2018). Rationalizing why some workers and firms choose informal employment and understanding how informality interacts with labor market policies are the goals of this paper. This paper contributes to the literature by proposing a theoretical model that answers both of these questions. In our empirical exercise, we study the connection between minimum wages and informality in Brazil, a developing country characterized by a large informal sector.

We use a framework, closely following Roy (1951) and Rosen (1986), to explain the heterogeneity in the type the employment – formal versus informal – and wages. Workers and firms bargain over the wage *and* the sector under which production will take place. This process generates an equilibrium distribution of employment, sector choices – formal and informal –, and wages that possesses the key features of both the Roy model and the compensating differentials model. We also show that the relative strengths of these two mechanisms are related to the bargaining power parameter: In the limit case in which the worker has all the bargaining power, the model collapses to the standard Roy model. On the opposite end of the spectrum, when the firm has all the bargaining power, wages feature only compensating differentials.

We show that informal sector workers can be characterized according to their second-best sector alternative (formality versus non-employment) and that, although these informal workers all share the same sector choice, they differ substantially in the way that they respond to labor market policies. As a result, the economy's response to a policy that increases the costs of informality will largely depend on the fraction of workers that belong to each group. Similarly, we show that a similar taxonomy can be constructed for formal workers: They can be characterized according to their second-best alternative (informality versus non-employment). The economy's response to policies that increase the costs of operation in the formal sector (such as an increase in taxes or the minimum wage) will largely depend on the type of jobs formal workers hold.

In our empirical exercise, we estimate the effects of the minimum wage using a nation-wide representative dataset of the Brazilian population known as "PNAD" for the years 2001-2005. We find that the minimum wage increases expected wages in the formal sector. On the other hand, the policy decreases the size of the formal sector. This reduction is induced by both the disemployment effects of the policy and the reallocation of workers to the informal sector as a result of the minimum wage.

Our results are related to the vast literature concerning the economic effects of the minimum wage (Card and Krueger (1994), Card (1992), Dube et. al. (2010), Lemos (2009), Sorkin (2014), Meer and West (2015), Aaronson (2001), Aaronson et. al. (2012), Brochu and Green (2012), MaCurdy(2015), Baker et. al. (1999), Flinn (2006), Tonin (2011), Jales (2017), Cengiz et. al. (2017), Heckman and Sedlacek (1981), Slichter (2015), Engbom and Moser (2017), Monras (2019), Saltiel and Urzúa (2018), and many others). Our results also relate to the literature concerning the phenomena of informality in developing countries and its relation to productivity (Galiani and Weinschelbaum (2012), Meghir, Narita, and Robin (2015), Rauch (1991), Matos and Ogura (2009), Ulyssea (2010, 2018), and many others). Lastly, our empirical exercise is closely related to the econometric literature that exploits bunching (Saez (2010), Kleven (2016), Jales and Yu (2017)) and discontinuities (Hahn et al. (2002), Calonico et al. (2014)) to identify the causal effects of policy interventions.

Our analysis has three main contributions: (i) We provide a novel taxonomy of formal workers/firms that extends the taxonomy typically used in the development literature for informal workers. We show that whether a formal worker will lose his job or move to an informal contract as a result of any policy that increases the costs of formality fundamentally depends on which class the worker/firm belongs according to our proposed taxonomy, and that bounds for the proportion of workers that belong to each of these types can be obtained using policy variation such as labor taxes or the minimum wage.[1](ii) We study the joint determination of employment, sector assignment (formal versus informal), and wages under a bargaining framework. This framework nest the standard Roy model in one extreme of the bargaining power space as a special case. Our analysis generates useful insights for when we should expect differences in wages of workers assigned to different sectors to identify sector productivity differentials, compensating wage differentials, or some weighted average of both. Our results can help to shed light on how to interpret the results of the literature that has investigated the causal effects of informality on wages (see, for example, Otero-Cortes, 2018). (iii) Lastly, we endow our theoretical framework with statistical assumptions on the shocks of the model in a way to arrive at a tractable likelihood function. We use this to study the effects of a particular policy that increases the costs of operation in the formal sector: the minimum wage. We show that under our statistical assumptions, the joint distribution of sector and wages that prevail under a minimum wage policy will collapse to the distribution proposed by Jales (2018). This result is, to the best of our knowledge, the first fully specified structural model for the labor market that rationalizes the statistical assumptions used in this reduced-form framework.

This paper is organized as follows: Section 2 discusses the model and its implications. Section 3 discusses the empirical strategy. Section 4 presents the results. Section 5 concludes.

2. A BARGAINING MODEL

2.1. *Environment*

Let worker's utility be given by: $U(l,s,w) = l \cdot (w - \epsilon + \eta_s)$, where w denotes the wage, $s \in \{0,1\}$ denotes the type of employment contract (one for the formal sector, zero

[1]Our model also includes a characterization of informal contracts. This characterization nests two of the leading explanations for the nature of the formal sector proposed in the literature. This characterization is similar (but less general than) the characterization proposed by Ulyssea (2018). Ullyssea's characterization nests all of the leading explanations for the nature of the formal sector in the context of a dynamic model. Since our model is static, it cannot include De Sotto's (1989) explanation, which is related to (red tape) entry costs to the formal sector.

otherwise), l denotes a binary indicator of whether the worker is employed, ϵ denotes the worker's outside option, and η_s denotes the amenity associated with employment at sector s. Let firm's profit be given by: $\Pi(l, s, w) = l \cdot (\alpha - w - \tau_s)$, where α denotes the worker's productivity, τ_1 the tax (and other costs) associated with formal sector employment, and τ_0 the costs of hiding this activity in the informal sector.

Workers and firms are heterogeneous with respect to $(\alpha, \epsilon, \tau_1, \tau_0, \eta_1, \eta_0)$. Assume that the economy consists of a large number of i.i.d. draws of the vector $(\alpha, \epsilon, \tau_1, \tau_0, \eta_1, \eta_0)$, where, for each draw, workers and firms behave as described below. This is implicitly the same as assuming that all labor markets are segmented (perhaps across narrowly defined city-occupation-industry cells), as in Lavecchia (2017), Lee and Saez (2008), and others. In practice, this essentially restricts the analysis to the short-run, so that workers and firms are not able to move across different markets or locations to bid for jobs or workers that operate in different markets.[2]

In a given sector-occupation-city cell, a (single) worker bargains with a firm. The worker's problem is to choose $\vec{l} \in \{0, 1\}$, that is, to choose whether to work or not, for any offer of the pair (w, s). The solution to this problem is characterized by a threshold associated with the worker's participation constraint $(U(w, s, l) \geq 0)$:

$$\vec{l} = \mathbb{1}\{w - \epsilon + \eta_s > 0\}.$$

Similarly, the firm's problem is to choose $\overleftarrow{l}$, that is, to employ the worker or not, for any given a pair of (w, s).[3] The solution to the firm's problem is also a threshold

[2] This assumption rules out some mechanisms that are important in the analysis of the effects of the minimum wage, such as labor-labor substitution – as in Cengiz et al. (2017) – and changes in labor market tightness – as in Flinn (2006)–. We discuss later how the results of the empirical exercise can be made robust to violations of this restriction.

[3] We focus on the decision of a single worker bargaining with a single firm. Thus, we think of every firm having at most a one employee. There are important relationships between firm size and informality, all of which we abstract in our discussion. An interesting framework that explicitly accounts for firm size in the context of informality can be found in Galiani and Weinschelbaum (2012).

associated with the firm's participation constraint ($\Pi(w,s,l) \geq 0$):

$$\overleftarrow{l} = \mathbb{1}\{\alpha - w - \tau_s > 0\}.$$

In general, there are multiple pairs of (w,s) such that labor supply equals labor demand, $\vec{l} = \overleftarrow{l}$, that is, there are many values for (w,s) that induce both worker and firm to agree on employment. To resolve this indeterminacy, assume that the equilibrium pair (w^*,s^*) is decided according to the solution of a Nash bargaining problem:

$$(w^*, s^*) = \arg\max_{(w,s)} (\Pi(l,s,w))^{1/2}(U(l,s,w))^{1/2}, \tag{1}$$

where w^* and s^* are the equilibrium wage and sector that prevail in the unconstrained case, that is, in the absence of any labor market institution such as the minimum wage. In the absence of a minimum wage policy, the bargaining process between worker and firm operates unconstrained. Thus, any wage that solves the maximization problem above, no matter how small, is valid. The solution to problem (1) is characterized as follows and is proved in Appendix A.

$$s^* = \mathbb{1}\{\eta_1 - \eta_0 > \tau_1 - \tau_0\}, \text{ and } w^* = \frac{1}{2}\left(\alpha + \epsilon - \tau_{s^*} - \eta_{s^*}\right). \tag{2}$$

The worker will be employed in the formal sector if and only if the net (aggregate) benefit is larger in the formal sector compared to the informal sector.[4] Also, we have that l^*, the equilibrium employment, has the property that $l^* = 1 \iff \min\{U(1,s^*,w^*), \Pi((1,s^*,w^*)\} > 0$, that is, the participation constraints of both worker and firm have to be met for employment to be equal to one. The equilibrium wage will be such that the total surplus from the transaction is split between worker and firm.

[4]Note that this solution has the intuitive property that an increase in tax rates (or a decrease in the cost of informality) induces the marginal workers to move to the informal sector.

Define w_1 (w_0) as the equilibrium wage a worker would get if he were employed in the formal (informal) sector. From the solution to the Nash Bargaining problem, we have:

$$w_1 = \frac{1}{2}\left(\alpha + \epsilon - \tau_1 - \eta_1\right), \text{ and } w_0 = \frac{1}{2}\left(\alpha + \epsilon - \tau_0 - \eta_0\right). \tag{3}$$

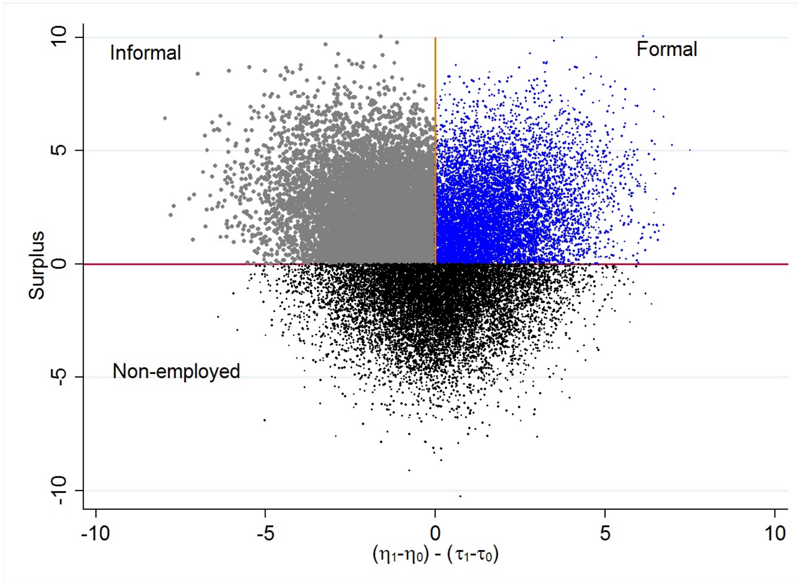
The decision of formal versus informal employment is endogenous. It is based on the opportunity costs of each state. In equilibrium, a worker will be employed in the formal sector if $\tau_1 - \eta_1 < \tau_0 - \eta_0$. If the inequality is reversed, the worker will be employed in the informal sector. We can then re-write w^* as $w^* = s^* w_1 + (1 - s^*) w_0$ as the equilibrium wage of a employed worker.

Figure 1 displays the choice of employment and sector for different workers. The horizontal line characterizes the geometric locus of workers and firms at the margin of indifference between employment and non-employment. The vertical line characterizes the geometric locus of firms and workers that are indifferent between producing in the formal or the informal sector. Workers in the upper right quadrant are employed in the formal sector, workers in the upper left quadrant are employed in the informal sector, and workers in the bottom half are non-employed.

In general, the informal sector will have both firms that could operate in the formal sector and firms that could not, given the current level of taxes and benefits. That is, the model encompasses two of the leading explanations for informality as special cases.[5] To characterize these different groups, consider the thought experiment of increasing enforcement. This is the case when $\tau_0 \to \infty$. In this situation, the costs of

[5] Ulyssea (2018) obtains a similar result in the context of a dynamic model. Ulyssea's result also incorporates a third view that associates informality with red tape and entry costs to the formal sector. We abstract from this issue since, in a static model, there is no distinction between the costs of formality and the costs of entering the formal sector. Another important distinction is that we only model informality of the employment relationship, not the firm's decision to register and pay profit taxes. In the terminology of Ulyssea, our model is about employment contract informality (the intensive margin), whereas Ullysea's model is about both the extensive margin of informality (the firm's registration choice) and also the intensive margin of informality (the nature of the employment contracts). For a more detailed discussion of these issues, see Ulyssea (2018).

Figure 1: Employment and sector choice



Note: Surplus is defined as $r^* = r_1 s^* + r_0(1 - s^*)$, where $r_s = \alpha + \eta_s - \epsilon - \tau_s$.
Source Simulated data from the model using Gaussian draws. See text for more details.

operating in the informal sector become prohibitively large. As a result, the reaction of the firms will be to either move to the formal sector or to go out of business. Whether one or the other option will be chosen depends on the surplus associated with the choice of operating in the formal sector. Thus, the response of the firm to an increase in enforcement will depend on the ranking of the surplus of being formal, informal, or non-participating before the policy of enforcement is introduced.

Parasite: $\lim_{\tau_0 \to \infty}(s^*, l^*) = (1, 1) \iff \alpha - \epsilon + \eta_0 - \tau_0 \geq \alpha - \epsilon + \eta_1 - \tau_1 \geq 0.$

Survival: $\lim_{\tau_0 \to \infty} l^* = 0 \iff \alpha - \epsilon + \eta_0 - \tau_0 \geq 0 \geq \alpha - \epsilon + \eta_1 - \tau_1.$

These equations show that some firms will react to the policy of increased enforcement by moving to the formal sector. The firms that would choose to do that are the ones in which $\alpha - \epsilon + \eta_0 - \tau_0 \geq \alpha - \epsilon + \eta_1 - \tau_1 \geq 0$. That is, the surplus in the informal sector (before enforcement is introduced) is higher than the surplus in the formal sector, which is higher than non-participating. In the terminology of La Porta and Shleifer (2008), these firms correspond to the "parasite" view of informal employment.[6]

[6]It is worthwhile to note that La Porta and Shleifer's taxonomy is originally designed to characterize informality of the firm – that is, the decision to register and pay taxes–, whereas we are

On the other hand, there are firms that will react to the policy of increased enforcement by going out of business. The firms that would choose to do that are the ones in $\alpha - \epsilon + \eta_0 - \tau_0 \geq 0 \geq \alpha - \epsilon + \eta_1 - \tau_1$. For these firms, the surplus (before enforcement is introduced) is larger than the surplus of non-participation, and the surplus of non-participation is larger than the surplus in the formal sector. These firms will react to increased enforcement by leaving the market. They, in the terminology of La Porta and Shleifer (2008), correspond to the "survival" view of informal employment.[7]

This result shows that a policy that increases enforcement will, in general, induce some workers to become formal. However, this will happen at a cost of inducing other workers to become unemployed. The geometrical locus that separates the parasite from the survival companies is given by the negative 45-degree line, the line that bisects the second and fourth quadrant. Figure 2 displays this relationship.

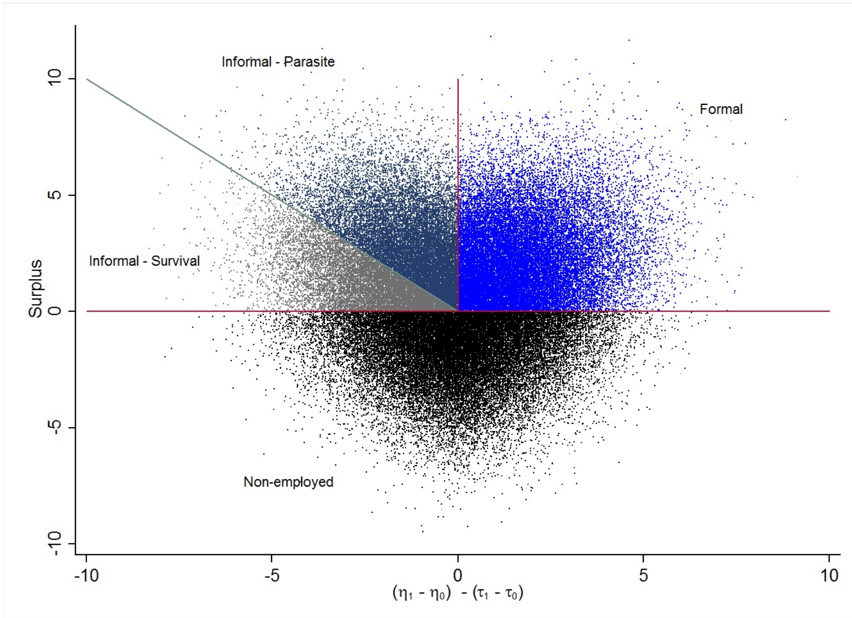
The model also allows for a similar taxonomy of *formal* firm-worker pairs. These firms can be divided according to their response to a change in the environment that increases the costs of operating in the formal sector. We will denote the firm (and worker pairs) that respond to an increase in τ_1 ($\tau_1 \to \infty$) by going out of business as "formal contracts of the first kind". For these firms, there is no surplus from operating in the informal sector. As a result, their next best option is not to operate at all. Alternatively, we will denote by "formal contracts of the second kind" the formal worker and firm pairs that respond by moving to the informal sector (when $\tau_1 \to \infty$). For these firms, the surplus of operating in the informal sector is positive. They choose to operate in the formal sector because at the current value of τ_1 the surplus is larger when they are formal.[8]

borrowing their definition and applying it to characterize informality of employment contracts. There are important distinctions between firm informality and employment informality and the interaction between these two phenomena can be economically relevant as well (Ulyssea, 2018). For the purposes of our model and empirical exercise, we will abstract completely from the process of firm informality and only focus on informality of employment contracts. So, whenever we refer to a informal firm in our discussion, we mean a firm-worker pair for which the employment contract is informal.

[7]For an example of an actual policy that closely mimics our thought exercise of complete enforcement, see Guzman (2017).

[8]The model predicts that attempts to increase the number of formal sector jobs by increasing the

Figure 2: The parasite and the survival view of Informality

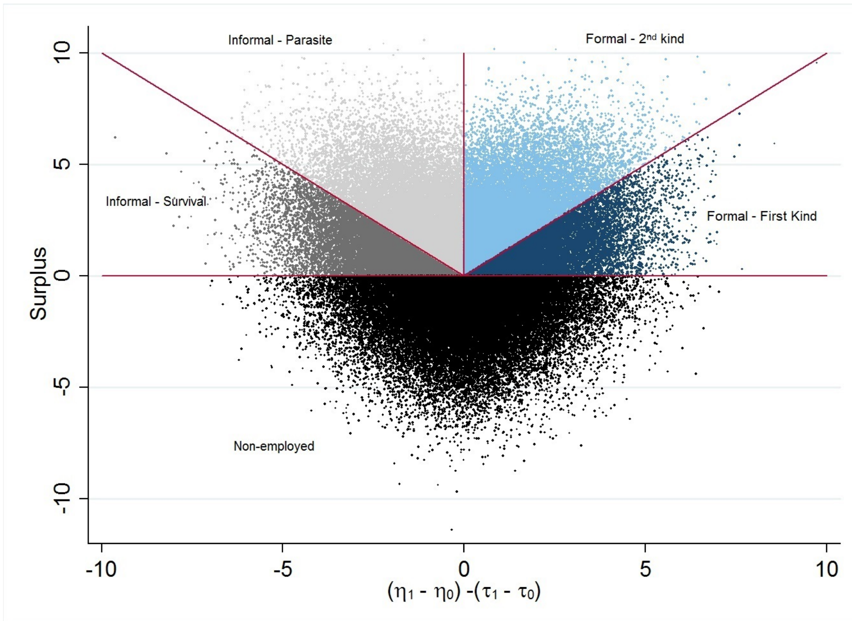


Note: Surplus is defined as $r^* = r_1 s^* + r_0(1 - s^*)$, where $r_s = \alpha + \eta_s - \epsilon - \tau_s$.
Source Simulated data from the model using Gaussian draws. See text for more details.

This characterization is helpful when thinking about the effects of the minimum wage, as the minimum wage policy introduces a constraint on firm-worker pairs operating in the formal sector. Employment relationships of the second kind will never respond to the policy by terminating the contract. The worker and firm only need to decide between staying in the formal sector and paying the minimum wage or to operating in the informal sector. On the other hand, workers and firms that belong to the other class – that is, those of employment of the first kind – they will never operate in the informal sector. Thus, their choice is limited to remaining in the formal sector and paying the minimum wage or terminating the contract. As one can see, in an economy characterized by the possibility of informality, the effects of the minimum wage will largely depend on how many contracts in the formal sector are characterized by parasite firms and how many are characterized by survival firms (and, naturally, how

costs of operating in the informal sector (such as strengthening enforcement) will invariably generate some employment losses, whereas reducing the costs of operating in the formal sector (by decreasing taxes) will move marginal workers from the informal to the formal sector *and*, in addition, create new jobs (Ulyssea, 2010).

Figure 3: The taxonomy of formal and informal employment



Note: Surplus is defined as $r^* = r_1 s^* + r_0(1 - s^*)$, where $r_s = \alpha + \eta_s - \epsilon - \tau_s$.
Source Simulated data from the model using Gaussian draws. See text for more details.

many workers are affected by the minimum wage). Figure 3 displays the geometric locus that separates these types of firms.

Formal employment of the first kind:

$$\lim_{\tau_1 \to \infty} l^* = 0 \qquad \iff \alpha - \epsilon + \eta_1 - \tau_1 \geq 0 \geq \alpha - \epsilon + \eta_0 - \tau_0$$

Formal employment of the second kind:

$$\lim_{\tau_1 \to \infty}(l^*, s^*) = (1, 0) \iff \alpha - \epsilon + \eta_1 - \tau_1 \geq \alpha - \epsilon + \eta_0 - \tau_0 \geq 0$$

2.2. *Relationship with Rosen and Roy models*

The model combines two distinct explanations for differences in wages across sectors: Compensating differentials and (Roy-model type of) self-selection. When $\eta_0 = \eta_1$, the worker is always employed in the sector where the wage draw is larger. That is, $s^* = \mathbb{1}\{w_1 > w_0\}$, as in the standard Roy-model. When $\tau_1 = \tau_0$, workers sort themselves based on their idiosyncratic valuation of amenities, that is, $s^* = \mathbb{1}\{\eta_1 > \eta_0\}$. Note that a mechanism similar to compensating differentials is present since $\frac{\partial w_1}{\partial \eta_1} < 0$, so the worker's equilibrium wage is decreasing in η_1, the valuation of the amenities associated with working in that sector.

This section further explores the relationship between the model and the canonical models of Roy and Rosen. It is interesting to see how the equilibrium wage and sectors respond to changes in the worker's bargaining power. In the baseline setup, for simplicity, we set the worker's and firm's bargaining power to 0.5. It is interesting, however, to consider the results when the bargaining power approaches the limits of one or zero.

Proposition 1 *When the bargaining power of the worker is one, we have that:*
(a) $w_s = \alpha - \tau_s$*. This, in turn, implies that* $w_1 - w_0 = \tau_0 - \tau_1$*. Furthermore, we have that* $\frac{\partial w_s}{\partial \alpha} = 1$, $\frac{\partial w_s}{\partial \tau_s} = -1$*, and* $\frac{\partial w_s}{\partial \eta_s} = 0$*. Furthermore, we have that* $w_1 - w_0 = \Pi(1,1,w) - \Pi(1,0,w)$*, so differences in assigned wages across sectors are productivity differentials.*
(b) If, in addition, $\eta_1 = \eta_0$*, then the joint distribution of* (w_1, w_0, s^*) *is indistinguishable from the standard Roy model, in which* $s^* = \mathbb{I}\{w_1 > w_0\}$*.*
(c) Instead, if $\tau_0 - \tau_1$ *is constant across workers and* $Cov(\alpha, s^*) = 0$*, we have that* $\mathbb{E}[w_1|s^* = 1] - \mathbb{E}[w_0|s^* = 0] = \mathbb{E}[w_1 - w_0] = \tau_0 - \tau_1 = \Pi(1,1,w) - \Pi(1,0,w)$*. That is, differences in expected wages identify productivity differentials.*

Proposition 1 shows that in the particular case in which the workers have all the bargaining power, then wages cease to reflect Rosen's (1986) notion of compensating differentials. In this case, a worker's wage will have no relationship with the amenities associated with the job because he can extract all of the surplus associated with better amenities into higher utility levels. As a result, wages only reflect differences in the technology of production across sectors. These factors remain relevant in determining the worker's wage because they affect the participation constraint of the firm, whereas the amenities, which only show up on the worker's utility, do not.

The second part of Proposition 1 shows that if workers do not perceive differences in amenities across sectors, then the bargaining model features a joint distribution of sector and wages that is indistinguishable from the standard Roy model, in which workers select the sector that provides them the highest wage ($s^* = \mathbb{1}\{w_1 > w_0\}$).[9]

PROPOSITION 2 *When the bargaining power of the worker is zero, we have that:*
(a) $w_s = \epsilon - \eta_s$*. This, in turn, implies that* $w_1 - w_0 = \eta_0 - \eta_1$, $\frac{\partial w_s}{\partial \alpha} = 0$, $\frac{\partial w_s}{\partial \eta_s} = -1$, $\frac{\partial w_s}{\partial \tau_s} = 0$*. Furthermore,* $w_1 - w_0 = U(1,1,w) - U(1,0,w)$*, so difference in the assigned wages across sectors are compensating differentials.*
(b) If, in addition, $\tau_1 = \tau_0$*, then the joint distribution of* (w_1, w_0, s^*) *is indistinguishable from a modified Roy featuring only "seemingly irrational selection", in which* $s^* = \mathbb{1}\{w_1 < w_0\}$.
(c) Instead, if $\eta_1 - \eta_0$ *is constant across workers and* $Cov(\epsilon, s^*) = 0$*, we have that* $\mathbb{E}[w_1|s^* = 1] - \mathbb{E}[w_0|s^* = 0] = \eta_1 - \eta_0 = U(1,1,w) - U(1,0,w)$*. That is, differences in expected wages identify compensating differentials.*

Proposition 2 shows that in the particular case in which the firms have all the bargaining power, then wages cease to reflect differences in productivity across sectors. In this case, a worker's wage will have no relationship with the worker's productivity α or the sector-specific productivity shock τ_s. Just as in the previous case, the intuition for this result is simple: When the worker has no bargaining power, then the firm manages to capture any surplus that is associated with higher productivity (in the form of general productivity α or sector-specific productivity τ_s). This is the case because when the firm has all the bargaining power, then wages are only affected by the variables that show up in the worker's participation constraint, namely the disutility of work ϵ and

[9]It is interesting to note that in the original work by Roy (1951) the worker needs to decide between *fishing* and *hunting* (and not between two activities in which there is another party, the firm owner, that the worker needs to negotiate with). It is implicitly assumed in the model that no one owns the lake or the forest. Thus, the original Roy model implicitly implies that the worker has all the bargaining power since no one can claim a stake of the worker's proceeds.

the sector-specific amenities η_s. In this setting, wages only reflect the opportunity cost of employment ϵ, and the differences in wages across sectors reflect only Rosen's notion of compensating differentials. Part (b) of Proposition 2 shows the curious result that if firms do not perceive productivity differences assigning the worker to different sectors ($\tau_1 = \tau_0$), then the bargaining model features a joint distribution of sector and wages that looks quite peculiar. In this setting, workers are always employed in the sector in which they earn the *least*. There is, however, an intuitive explanation for this result: When the firm has all the bargaining power, and there are no differences in the production technology across sectors, then the sector choice that maximizes the surplus is the one in which workers are assigned to the sector that they like the most (the one with the highest value for the amenities). Since workers value the amenities, the corresponding wage that respects the worker's participation constraint is smaller. Thus, workers go to the sector that they like the most, and, because of that, they end up in the sectors where they earn the least ($s^* = \mathbb{1}\{w_1 < w_0\}$). These results show that, under the limit case of zero bargaining power for the worker, the relationship between wage differentials and the willingness to pay for amenities approach the one implied by Rosen's compensating differential model. On the other extreme, the relationship between wage differential and amenities does not reflect differences in willingness to pay for amenities. It instead reflects productivity differentials across sectors. When the bargaining power is in an interior point of the [0,1] interval, then wage differentials will partially reflect compensating differentials and partially reflect differences in unobserved productivity (or self-selection, as in Roy(1951)).[10,11]

[10]When the bargaining power of the worker is υ and $Cov(\eta_1 - \eta_0, \tau_1 - \tau_0) = 0$, then $Var[w_1 - w_0] = \upsilon^2 Var(\eta_1 - \eta_0) + (1-\upsilon)^2 Var[\tau_1 - \tau_0]$. Thus, the relevance of unobserved heterogeneity of productivity across sectors as an explanation for the differences between potential wages w_1 and w_0 is dictated by the variance of the differences in productivity, the variance of the differences in the valuation of amenities, and the worker's bargaining power. If $Var[\tau_1 - \tau_0] \approx Var[\eta_1 - \eta_0]$, then the fraction of the variance of the causal effect of sector on wage ($w_1 - w_0$) explained by Roy's mechanism is given by $\frac{\upsilon^2}{\upsilon^2 + (1-\upsilon)^2}$. That is, when the variance of taste differentials is similar to the variance of productivity differentials, then the relative importance of Roy's mechanism when compared to Rosen's mechanism is driven *entirely* by the bargaining power of workers and firms. For a paper that focuses on this type of decomposition in a different context, see Taber and Vejlin (2016).

[11]It is also interesting to note that it is possible for wages to be decreasing in the valuation

2.3. *The Minimum Wage*

Suppose a minimum wage policy is introduced in this economy. Assume that the policy is only binding in the formal sector; that is, if a worker is employed in the formal sector, his wage must be above the minimum wage, but if the worker is employed in the informal sector his wage can be greater or smaller than the minimum wage. Denote by $(\tilde{w}, \tilde{s})$ the pair of wage and sector that would prevail in this economy in the presence of the minimum wage.

The minimum wage policy introduces a new constraint in the Nash bargaining problem: $\mathbb{1}\{w < m\}sl = 0$, where m is the minimum wage level. This constraint simply states that it cannot be the case that $s = 1$, $l = 1$, and $\mathbb{1}\{w < m\} = 1$. The worker must be either informal, or unemployed, or, if he is employed and formal, it has to be the case that his wage is greater than or equal to the minimum wage. The introduction of the minimum wage changes the equilibrium joint distribution of sector and wages. This equilibrium can be characterized by solving the constrained Nash bargaining problem.[12]

The new equilibrium has the property that if in the absence of the minimum wage, the worker's wage was larger than the minimum wage ($w^* > m$), then the worker earns exactly the same wage once the policy is introduced. This is straightforward to see since if the constraint imposed by the minimum wage is not binding for (l^*, s^*, w^*) – the triplet employment, sector, and wage that solves the unconstrained Nash Bar-

---

of the amenities of the job even in a setting in which cross-market arbitrages are ruled out. The equilibrium notion in Rosen's model is one in which wages are decreasing in the value of amenities because workers can arbitrage (moving from the bad jobs to the good ones) and, as a result, the equilibrium wages will reflect the differences in the valuation of the amenities associated with the job (for the marginal worker). In this setting, workers cannot arbitrage across markets, but even still, wages will reflect the valuation of the amenities. A worker's wage would be decreasing in η even if there were only one sector in this economy, so no arbitrage is ever feasible. This happens because amenities imply a higher surplus for the match (profit plus utility levels). This extra surplus will be bargained over between the parties, and, as long as the worker is not able to fully capture the surplus, part of it will be transferred to the employer. This transfer takes the form of a decrease in the wage.

[12] In Appendix B we discuss in greater detail the welfare implications of the minimum wage in this model.

gaining problem – then it must be the case that (l^*, s^*, w^*) also solves the constrained optimization problem.

For the workers whose constraint $\mathbb{1}\{w < m\}sl = 0$ is binding when evaluated at the equilibrium wage, sector, and employment, the solution will be different than it would be in the absence of the minimum wage. Define the Nash Product by $B(l, s, w) = (\Pi(l, s, w))^{1/2}(U(l, s, w))^{1/2}$. The constrained solution to the Bargaining problem is given by: $w = m$, $s = 1$, and $l = 1$ if $\max\{B(1, 1, m), B(1, 0, w_0), B(0, \cdot, \cdot)\} = B(1, 1, m)$. This means that the worker remains employed in the formal sector and earns the minimum wage if the Nash product of this action is still positive (so it dominates being fired) and also it is greater than the Nash product obtained when employing him in the informal sector assigning him to the optimal informal sector wage. For later use in our empirical exercise, we denote the proportion of low wage formal workers that "bunch" at the minimum wage by $\pi_m^{(1)}$. This is an object we hope to estimate from the data.

Conversely, the worker will move to the informal sector if $\max\{B(1, 1, m), B(1, 0, w_0), B(0, \cdot, \cdot)\} = B(1, 0, w_0)$, that is, if the costs of moving the worker to the informal sector are smaller than keeping him in the formal sector and changing his wage. In this case, we have that $\tilde{w} = w_0$; that is, the worker will earn in the presence of the minimum wage the wager associated with his assignment to the *informal sector*, w_0. Lastly, if both $B(1, 1, m)$ and $B(1, 0, w_0)$ are negative, then the constrained solution to the bargaining problem will be to let $\tilde{l} = 0$, that is, the worker becomes unemployed. This will be the case if the worker's cost of informality is high and $\Pi(1, 1, m)$ is negative, that is the (net of taxes and opportunity costs) productivity of the worker is below the minimum wage. We denote the fraction of low-wage formal workers that move to the informal sector as a result of the policy by $\pi_d^{(1)}$. Lastly, we let the remaining fraction of low-wage formal workers that end up losing their jobs as a result of the minimum wage policy by $\pi_u^{(1)}$.

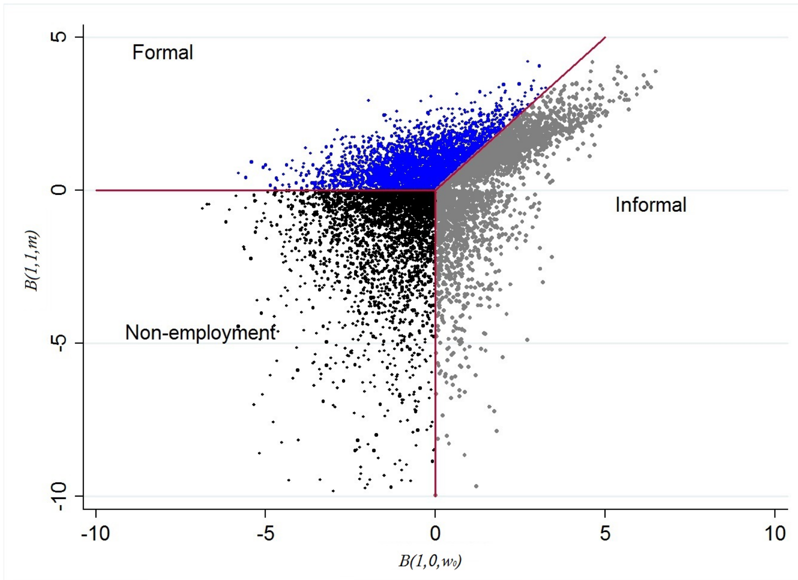
Figure 4 displays the responses of different formal sector workers for which the minimum wage constraint is binding. The southeastern corner consists of workers whose Nash product in the informal sector is still positive and larger than the Nash product evaluated at formal employment at the minimum wage. These workers move to the informal sector as a response to the minimum wage policy. As shown in Figure 3, all these workers are in formal employment contracts of the second kind. The northwestern corner consists of workers for which the Nash product when assigned to the formal sector at the minimum wage $B(1, 1, m)$ is positive and larger than the Nash product when assigned to the informal sector $B(1, 0, w_0)$. These workers remain formal and earn the minimum wage. These workers may belong to either formal-parasite or formal-survival firms. The southwest corner consists of the workers for which both the Nash product when assigned to the formal sector at the minimum wage $B(1, 1, m)$ *and* the Nash product when assigned to the informal sector $B(1, 0, w_0)$, are both negative. These workers lose their jobs once the minimum wage is introduced. All these workers are, as shown in Figure 3, in formal employment contracts of the first kind. The red lines characterize the geometric locus of the boundary of indifference between these choices.

The difference in the surplus of workers and firms when operating in the formal and informal sector is given by $\Delta = \frac{1}{2}(\eta_1 - \eta_0 - \tau_1 + \tau_0)$. The presence of Roy's comparative advantage (or unobserved heterogeneity) and Rosen's compensating wage differentials lead workers and firms to get distinct rents when they operate in different sectors. That implies that workers that end up moving to the informal sector due to the presence of the minimum wage could have utility levels that are, in principle, quite different than the wages that they would have in the formal sector. Our next result, however, shows that for small values of m, formal and informal sector surpluses must be approximately the same for the workers that move. Consider the effect of introducing a "small" minimum wage that approaches the lowest wage equilibrium in the formal sector; that is, m is slightly above $\inf w_1$. For simplicity, assume $\inf w_1 = 0$.

PROPOSITION 3 *Let c denote some positive constant, we have:*

Figure 4: Formal workers' response to the minimum wage

Employment and sector choice



Note: $B(l, s, w) = \Pi(l, s, w)^{0.5} U(l, s, w)^{0.5}$.

(a) For an arbitrary small $c > 0$*, if the minimum wage* $m \leq c^2$*, then* $\Pr\left[\Delta \leq c \middle| \max\{B(1,1,m), B(1,0,w_0), B(0,\cdot,\cdot)\} = B(1,0,w_0)\right] = 1$.

(b) For all m*,* $\lim_{c \downarrow 0} \Pr\left[\Delta \leq c \middle| \max\{B(1,1,m), B(1,0,w_0), B(0,\cdot,\cdot)\} = B(1,1,m)\right] = 0$.

(c) For all m*,* $\lim_{c \downarrow 0} \Pr\left[\Delta \leq c \middle| \max\{B(1,1,m), B(1,0,w_0), B(0,\cdot,\cdot)\} = B(0,\cdot,\cdot)\right] = 0$.

(d) For an arbitrary small $c > 0$*, if the minimum wage* $m \leq c$*, then* $\Pr\left[U(1,1,w_1) + \Pi(1,1,w_1) \leq c \middle| \max\{B(1,1,m), B(1,0,w_0), B(0,\cdot,\cdot)\} = B(0,\cdot,\cdot)\right] = 1$.

Proposition 3 is proved in Appendix B, Section 2.2. Part (a) implies that the probability that the difference in workers' utility between the formal and informal sectors (i.e., $\Delta = U(1,1,w_1) - U(1,0,w_0)$) is close to zero, conditional on switching from the formal to the informal sector as a result of a small minimum wage is 100%. In other words, for small values of m and for the subset of workers that move to the informal sector due to the minimum wage, we have that $U(1,1,w_1)$ is approximately the same as $U(1,0,w_0)$. As a result, the welfare cost of such a movement for these worker-firm pairs is negligible. Parts (b) and (c) show that the same probability is zero for the workers that bunch at m and also for those that lose their jobs. Part (d) shows that, when m is small, if the result of the minimum wage policy is to induce a worker-firm pair to dissolve the employment relationship, then the surplus of this employment relationship must be small.[13] The next proposition summarizes the expected welfare change of formal workers due to the minimum wage.

Proposition 4 *When the minimum wage is small, the change in formal worker's expected welfare is approximately given by $m \times \pi_m^{(1)}$ times the fraction of formal workers for which the policy is binding. This object can be bounded above by the minimum wage level times the fraction of formal workers that are observed with wages equal to the minimum wage. The upper bound coincides with the actual welfare approximation if the minimum wage generates no unemployment.*

Proposition 4 immediately follows from parts (a) and (d) of Proposition 3. Formal workers who are affected by the minimum wage react in one of three ways: (i). shift to informal sector; (ii) bunch at the minimum wage; and (iii) become unemployed. Part (a) and (d) of Proposition 3 imply that at a small minimum wage, the expected welfare change associate with reactions (i) and (iii) are close to zero. Therefore, the expected welfare change comes solely from workers with reaction (ii), that is, those who bunch at the minimum wage.

[13]Taken together, these results show that, if one only cares about the worker's welfare and no value is placed on the loss of labor tax revenues that comes out of the displacement of formal workers to the informal sector due to the minimum wage policy, then the optimal minimum wage in this economy is larger than zero. This is the case because both the rationing of jobs (unemployment effects) and the reallocation of jobs (movements to the informal sector) that the policy induces are both efficient in the sense of Lee and Saez (2012). Thus, the policy has a first-order welfare benefit for the workers that bunch at m and second-order welfare costs for those that do not.

3. EMPIRICAL STRATEGY

One key challenge in bringing this model to the data is that we do not typically observe certain objects, such as the value of time at home (ϵ), or the worker's valuation of formal job amenities (η_1). In this section, we discuss the simplifying restrictions we must impose on the model's structure so we obtain a tractable likelihood function that can be successfully computed when we only observe data on the pair of sector and wage, but not on the fundamental shocks that drive the model ($\alpha, \epsilon, \tau_1, \eta_1, \tau_0, \eta_0$). We show that, under certain restrictions, the model collapses to a two-sector structural version of the model used by Meyer and Wise (1983) to evaluate the effects of the minimum wage in the USA. We warn beforehand that although we believe that most of these restrictions are justified, one fruitful avenue for future research would be to attempt to estimate the model in a setting in which the econometrician observes a richer set of drivers of the variation in wage and sector choice, which would render disposable some of the restrictions we impose below.

Assumption 1 (Productivity distribution) *The worker's productivity α is drawn from a log-normal distribution.*

The assumption of log-normality is commonly invoked to model wage data (Meyer and Wise (1983), Laroque and Salanié (2002), and many others). Roy (1950) developed theoretical arguments that suggest that productivity should approximately follow a log-normal distribution.

Assumption 2 *Let u_s be the orthogonal projection of $\epsilon - \eta_s - \tau_s$ on α and β_s be the corresponding projection coefficient. The variance of u_s is small in the following sense: There exists two positive numbers t and k such that,* $\Pr[|\frac{u_s}{\alpha(1+\beta_s)}| > t] \leq k$.

This assumption states that the distribution of wages is, up to some location and scale shifts, *approximates* the distribution of worker's productivity. A more detailed description about the projection can be found in Appendix C, Section 3.1. Importantly, this assumption does not impose that the remaining forces in the model, such as the value of amenities, taxes, and time at home, are necessarily small. It allows workers to be out of the labor force ($\epsilon \to \infty$), it allows for some workers to arbitrarily large costs of formality or informality. It also allows amenities (η) and costs (τ) to be a non-trivial fraction of the worker's productivity or wages. What the assumption restricts is the probability that the component of these costs that varies independently of the worker's productivity becomes too large. Intuitively, what it states is that when we look at workers at quantiles far apart on the wage distribution, we are inclined to believe that there is a high likelihood that such large differences in earnings are due to differences in the marginal product of labor. The most extreme opposite of this assumption would be to believe that workers with large differences in earnings must have similar productivities but large differences in the tax treatments of their occupations (differences in τ) or large differences in the non-pecuniary aspects of the job (η). We stress that we do not bound how large η, τ, or ϵ can be, so workers with prohibitively large costs of formality or informality are not ruled out by this assumption, neither are workers that are out of the labor force.[14] In that sense, when we look at the distribution of wages and observe workers with drastically different wages, we do not assume that these differences should be immediately attributed to large differences in the valuation of amenities, neither to differences in their costs of operating in the formal/informal sector.[15]

Assumption 3 (Common-Support) *In the absence of the minimum wage, the sup-*

[14]For example, if the costs of formality for workers in a similar occupation are the sum of a constant fraction of their productivity plus a firm-worker specific costs of bookkeeping, then Assumption 2 implies that the distribution of worker-firm specific costs of formality (the bookkeeping costs) are small relative to the distribution of worker's productivity.

[15]Together, Assumptions 1 and 2 stated above place strong restrictions in the shape of the joint distribution of sector and wages, as we demonstrate in Proposition 5 below. As a result, these assumptions are partially testable. In particular, if they are severely misspecified, the model fit will likely be poor. Thus, an alternative way to judge the quality of these assumptions is to check whether the joint distribution of sector and wages implied by these assumptions can reasonably fit the observed data. We show in our empirical application that this is indeed the case for the Brazilian labor market, although we cannot say that this must be the case in other settings.

port of w_s *is* $\mathbb{R}^+$ *for both sectors.*

This assumption states that, without the minimum wage, the effective lower bound for both wage distributions is zero. This assumption is standard in the policy evaluation literature. The key implication of this assumption is that, in the absence of the minimum wage, the conditional probability of formality given the wage never touches zero or one. If this assumption were violated, in the absence of the minimum wage, one could predict with 100% accuracy the likelihood that a worker is formal or informal just by knowing his latent wage, which would make the conditional probability model degenerate. We believe that this is not a controversial assumption. In Appendix C, Section 3.1, we show that, in the absence of such an assumption, we only need to add one extra parameter to the model. The added parameter characterizes the lower end of the support of the formal sector latent wage distribution.[16]

3.1. *Implications*

Under the assumptions 1, 2, and 3 presented in the previous section, we can characterize the joint distribution of sector and wage both in the absence and in the presence of the minimum wage. In this section, we show this characterization and how it implies a particular structure for the effects of the minimum wage, which can be thought of as a two-sector extension of the empirical framework proposed by Meyer and Wise (1983).

Given that our interest lies in estimating the causal effect of the minimum wage, in the following discussion that follows it is useful to employ the Rubin potential outcomes notation. In the presence of the minimum wage, a worker i is characterized by a wage $W_i(1)$ and a sector $S_i(1)$, which is equal to one if the worker is employed in the formal sector and zero otherwise.

[16]We stress that in the presence of the minimum wage, the support of the wage distributions will likely not be the same, since the formal sector wage distribution has a lower bound at m, whereas the informal sector wage distribution has a lower bound at zero. Our assumption is about the support of these wage distributions in the absence of a price floor.

Also, let the pair $(W_i(0), S_i(0))$ denote the counterfactual – or latent – wage and sector that prevail in the absence of the minimum wage. Finally, define $F_0(w)$ $(f_0(w))$ as the c.d.f (p.d.f) of latent wages $W(0)$ and $F(w)$ $(f(w))$ as the c.d.f (p.d.f) of observed wages ($W_i(1)$ or, using shorter notation, W_i). We assume that the econometrician observes a random sample of i.i.d. draws of the pair $(W(1), S(1))$, that is, the econometrician only observes the data in the presence of the policy.

Proposition 5 (Latent wage distribution) *Suppose Assumptions 1 to 3 hold. In the absence of the minimum wage, the density of potential wages in the formal sector* $f_{w_1}(x)$ *and in the informal sector* $f_{w_0}(x)$ *follow a log-normal distribution. In addition, if* $(\log w_s, \Delta)'$ *approximately have a joint normal distribution, then the density of log-wages for those that choose to work in each sector – that is, the formal sector density* $f_{\log w_1}(x|S(0) = 1)$ *and the informal sector density* $f_{\log w_0}(x|S(0) = 0)$ *– follow skew-normal distributions. The unconditional log-wage distribution* $f_{\log w}(x)$ *follows a mixture of skew-normals. In addition, if* $S(0)$ *is independent of* α*, then this mixture of skew-normals is well-approximated by a mixture of normals. Instead, if* $w_1 \approx w_0$*, then this mixture of skewed-normals collapse to a (single component) normal.*

The exact form of wage distributions mentioned in Proposition 5 and its proof can be found in Appendix C, Section 3.1. Proposition 5 specifies the shape of the distribution of wages, both across sectors and also unconditionally, under different restrictions. Our parametric family nest well-known parametric families used to model the distribution of earnings. For example, when $\log w_1 \approx \log w_0$ and also $S(0)$ is independent of α, then the distribution of wages collapses to the log-normal wage distribution that has been used by Roy (1950). If we relax these two constraints, then the wage distribution we obtain is identical to the canonical form of the Roy (1951) model of self-selection. Finally, if we drop the common-support assumptions, the wage distribution becomes a mixture of skewed *and shifted* log-normals. Defining θ_0 to be the parameters of this distribution and f_0 to be the latent density of wages, we have that, $f_0(w) = f_0(w; \theta_0)$.

The advantage of using a parameterized functional form for the latent distribution of wages is that it allows for maximum likelihood estimation of the model parameters. On the other hand, the robustness of the empirical results to deviations from log-normality is lost. Thus, there is a trade-off. In our empirical application, we show that the Brazilian log-wage distribution seems to be well approximated by a normal distribution, so even our most restrictive assumption provides an accurate approximation of the target distribution. The parametric assumption does not seem to impose a significant loss of credibility in this particular exercise.

Proposition 6 (Conditional probability of the (latent) sector given the wage) *The conditional distribution of the latent sector, given the latent wage, belongs to the logit parametric family* $\{\Lambda(w, \delta) : \delta \in B \subset \mathbb{R}^k\}$*. That is,* $\Pr[S(0) = 1|W(0) = w] = \Lambda(w, \delta_0)$ *for some* $\delta_0 \in B$*.*

This result states that the conditional probability of formality given the wage, in the absence of the minimum wage policy, can be approximated using a logit model, see the Appendix C, Section 3.2 for details. In our empirical application, we find that even a trivial model that forces the coefficient of the wage in the logistic regression to be zero would give a reasonable fit to the data.

To complete the characterization of the joint distribution of latent and observed wages, we need to characterize the form in which the minimum wage affects the joint distribution of sector and wages, both above and below the minimum wage level. To do that, we use the results from our theoretical model. For completeness, we summarize the key model implications we use to derive the likelihood function in the following remarks:

Remark 1 (Limited spillovers) *Workers whose latent wages would be above* k *times the minimum wage are not affected by the policy. That is,* $W(1) = W(0)$ *and* $S(1) = S(0)$ *when* $W(0) > km$*, for a known* k *greater than or equal to one.*

This remark only restates the fact that the constraint imposed by the minimum wage in this labor market is not binding for workers that earn more than m. Thus, as a result, there will be no economic spillover effects of the minimum wage above km for any $k \geq 1$. However, this result still allows the upper part of the wage distribution to be affected by the rescaling associated with the inflows and outflows of workers that will occur as a result of the policy.

The structural model presented in Section 2.1 implies that one could choose $k = 1$, since the model predicts that $W(1) = W(0)$ whenever $W(0) < m$. However, this is a simple consequence of the assumed absence of linkages between the labor market of low-wage and high-wage workers in our model. In other words, the absence of spillovers we obtain is close to an apriori assumption of the model. In many other models (see, for example, Engbon and Moser, 2018), the minimum wage would induce spillover effects higher up on the wage distribution. Moreover, there is evidence that these effects are non-trivial (Engbon and Moser, 2018).[17] Thus, we highlight here that we only need spillovers to become small as we look further up on the wage distribution, which is an assumption justified in most if not all models in which the minimum wage generates spillover effects. In our baseline empirical results, we will assume away any spillover, which is the same as setting k to be one. In Appendix D, we discuss how one can estimate the model parameters allowing for some degree of spillovers higher up on the wage distribution and discuss the results we obtain using this more robust assumption.

Remark 2 (Minimum wage effects' structure (strong characterization)) *For wages below the minimum wage* ($W(0) < m$)*, we have the following: If* $S(0) = 0$*, then* $S(1) = S(0)$*. Additionally, with probability* $\pi_d^{(0)}$*, – which is potentially a function of the worker's latent wage–, the wage* ($W(1) = W(0)$) *continues to be observed. With the complementary probability* $\pi_m^{(0)} = 1 - \pi_d^{(0)}$*, the worker earns the minimum wage* ($W(1) = m$)*.*

[17] The economic mechanisms commonly invoked to explain how the minimum wage can affect workers that would already earn more than the minimum wage are low-skill/high-skill labor substitution (Teulings (2000)) and search externalities (Flinn (2006), Engbom and Moser (2017)).

If $S(0)=1$, then with probability $\pi_d^{(1)}$, – which is also potentially a function of the worker's latent wage– the wage ($W(1)\approx W(0)\xi$) continues to be observed, meaning that the worker successfully transitions from the formal to the informal sector and thus will earn his corresponding informal sector's wage.[18] *In this case, the observed sector will be $S(1)=0$, which differs from the latent sector. With probability $\pi_m^{(1)}$, the worker earns the minimum wage ($W(1)=m, S(1)=1$). With the complementary probability ($\pi_u^{(1)}=1-\pi_d^{(1)}-\pi_m^{(1)}$), the worker becomes unemployed ($W(1)=\cdot, S(1)=\cdot$).*

Note that we allow here the possibility that informal workers will also "bunch" at m when the minimum wage is introduced. We abstract from these considerations in our theoretical exercise, but we accommodate for this regularity in our empirical exercise.[19]

3.2. *Estimation*

Collecting the results of the previous section, we obtain a tractable expression for the likelihood of the data under the minimum wage policy. Let $\Theta\equiv(\theta,\delta,\pi)$ be the entire vector of model parameters, that is, those governing the latent distribution of wages, the conditional probability of sector given wages and minimum wage effects. Define the likelihood of observing a pair (w,s) given the minimum wage level m and model parameters Θ as $L(W(1)=w,S(1)=s|\Theta)=\Pr[S(1)=s|W(1)=w;\Theta]f(w|\Theta)$.
Given that $\log(L(W(1)=w,S(1)=s|\Theta)=\log\Pr[S(1)=s|W(1)=w;\Theta]+\log f(W(1)=w|\Theta)$, we can define the maximum likelihood estimator of the model parameters as $\widehat{\Theta}=\arg\max_{\Theta}\frac{1}{N}\sum_i^N\log L(w_i,s_i|\Theta)$.
The numerical optimization of the likelihood function can be simplified using a three-

[18] The parameter ξ that appears on the expression $W(1)\approx W(0)\xi$ measures the expected change on a formal worker's wage when he moves to the informal sector because of the minimum wage. ξ corresponds to the ratio of the means of the latent distributions in the informal and formal sectors.

[19] These probabilities are reduced-form parameters that characterize how the minimum wage affects wages and formality in the economy. These objects have, under the assumptions of the model, internal validity to assess the effects of a particular minimum wage level and the effects of small changes on it, but they are not invariant to changes in the economy, so they are subject to external validity concerns and the Lucas' critique.

step procedure. First, estimate the parameters of the latent wage distribution by considering only the values above the minimum wage:

$\widehat{\theta} = \arg\max_{\theta} \frac{1}{N}\sum_i^N \mathbb{1}\{w_i > m\} \log f(w_i|w_i > m; \theta)$.

The likelihood function in this first step closely resembles the likelihood from a Tobit regression model (Tobin, 1958). To estimate the conditional probability of the latent sector given the wages, one can run a logit regression *using only wages above the minimum wage*: $\widehat{\delta} = \arg\max_{\beta} \frac{1}{N}\sum_i^N \mathbb{1}\{w_i > m\} \log \Pr[s_i|w_i; \delta]$.

Finally, maximize the likelihood function over the subset of parameters that remains to be estimated π using the full sample: $\widehat{\pi} = \arg\max_{\pi} \frac{1}{N}\sum_i^N \log L(w_i, s_i|\widehat{\theta}, \widehat{\delta}, \pi)$.

This procedure yields consistent estimates because the density of wages for values above the minimum is merely a function of a subset (θ) of the parameter vector (Θ) and the fact that the true parameter θ_0 that governs the shape of the density of *latent* wages is the argument that maximizes $\mathbb{E}[\log f(W(1)|W(1) > m; \theta)]$.[20] The same holds for the conditional probability of sector given the wage: The conditional probability of observed sector given the wage, for values above the minimum wage, is only a function of δ and the true parameter δ_0 that governs the relationship between *latent* sector and wages is the argument that maximizes $\mathbb{E}[\log(\Pr[S(1)|W(1) > m; \delta])]$. In this case, estimation is simple: In the first step, one merely needs to estimate a truncated regression of wages on a constant for values above the minimum wage. Then, in the second step, one needs to estimate a logit regression of sector on wages, using only values above the minimum wage, as before. Only in the last step is the entire likelihood function numerically optimized to recover π. Efficiency can then be improved by using these estimates as initial values for the maximum likelihood estimator $\widehat{\Theta}$.

In Appendix D, we discuss in detail how one can modify these estimators to obtain robustness to minimum wage effects higher up on the wage distribution (spillovers) and how to obtain robustness to misspecification of the minimum wage effects on the

[20]Strictly speaking, this holds whenever ξ is smaller than one, that is, *on average*, latent wages in the formal sector are larger than on the informal sector. In the general case, one needs to condition on wages larger than $m \times \xi$.

Table I: Descriptive Statistics

Parameter	2001	2002	2003	2004	2005
$E[W]$	5.92	5.98	6.08	6.15	6.25
$Sd[W]$	0.73	0.73	0.71	0.70	0.67
Skewness[W]	0.37	0.31	0.22	0.19	0.19
Kurtosis[W]	4.28	4.52	4.66	4.76	4.75
$Pr[W = m]$	0.09	0.12	0.13	0.12	0.16
$Pr[W < m]$	0.06	0.07	0.08	0.08	0.08
$q^{80}[W]$	6.41	6.54	6.58	6.68	6.68
$q^{20}[W]$	5.30	5.38	5.48	5.56	5.70
$E[W\|S = 1]$	6.07	6.13	6.23	6.31	6.39
$E[W\|S = 0]$	5.57	5.63	5.71	5.77	5.88
$Sd[W\|S = 1]$	0.66	0.65	0.62	0.61	0.58
$Sd[W\|S = 0]$	0.77	0.78	0.77	0.75	0.74
$q^{80}[W\|S = 1]$	6.55	6.62	6.68	6.75	6.80
$q^{20}[W\|S = 1]$	5.52	5.58	5.70	5.77	5.86
$q^{80}[W\|S = 0]$	6.11	6.21	6.21	6.25	6.40
$q^{20}[W\|S = 0]$	5.19	5.19	5.30	5.30	5.30
$Pr[S = 1]$	0.70	0.70	0.71	0.71	0.72
$Pr[S = 1\|W < m]$	0.08	0.08	0.09	0.04	0.03
$Pr[S = 1\|W = m]$	0.58	0.53	0.63	0.63	0.62
$Pr[S = 1\|W > m]$	0.76	0.78	0.79	0.79	0.81
m	5.19	5.30	5.48	5.56	5.70

Note: Nominal wages in Brazilian R$.

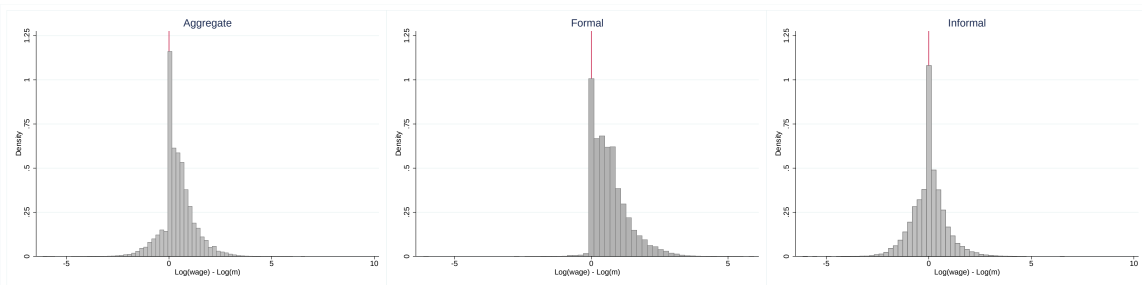
bottom part of the wage distribution.

4. EMPIRICAL APPLICATION: THE EFFECTS OF THE MINIMUM WAGE IN BRAZIL

4.1. *Descriptive Statistics*

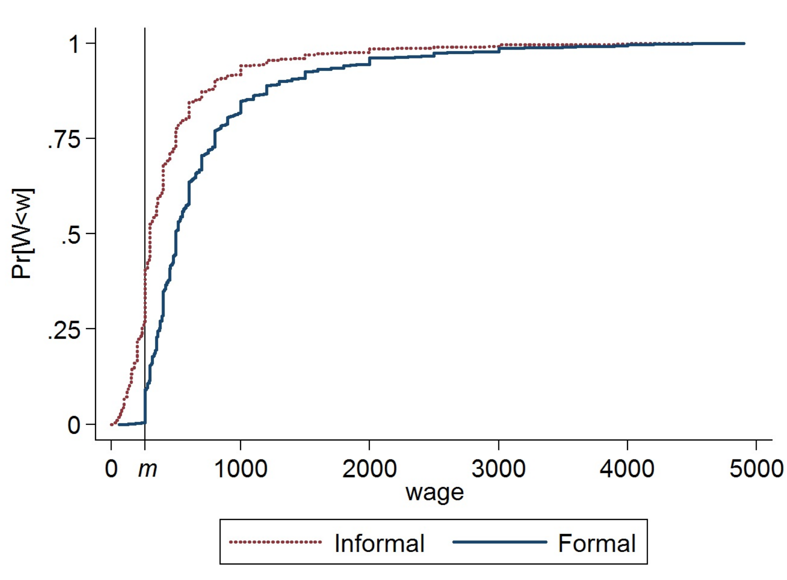
To evaluate the impact of the minimum wage in Brazil, we used the PNAD household survey pooling years from 2001 to 2005. PNAD is an acronym for the Portuguese name of the survey, which can be translated as "Nationwide Household Sample Survey". These data, which are representative of the Brazilian population, are collected yearly by the IBGE, a Brazilian statistical agency. Workers who do not report wages, workers who work in the public sector, and workers who are older than 60 years of age or younger than 18 were removed from the sample. Additionally, workers who report monthly wages above R$5000 were removed from the sample, which excludes the upper 1.15% of the wage data.

Figure 5: Wage distribution



Note: Data from years 2001 to 2005. Log-wages normalized around the minimum wage level.

Figure 6: Empirical CDFs by Sector



Note: Year 2004.

Table I and Figures 5 and 6 present some empirical facts concerning the joint distribution of sectors and wages for Brazilian data from the 2001–2005 period. Regarding Table I, we observe that expected wages are higher during the later years, which are characterized by higher minimum wage levels. Wages in the formal sector are, on average, higher than wages in the informal sector. The informal sector comprises a large share of the aggregate economy – approximately 28% based on these data. The probability that a wage is equal to the minimum wage ($\Pr[W = m]$) ranges from 8 to 14%. The proportion of workers who receive wages below the minimum wage $\Pr[W < m]$ ranges from approximately 6 to 8%. The probability of working in the formal sector as a function of the wage is discontinuous. It is approximately 79% for values above the minimum wage, 59% at the minimum wage and virtually zero below it.

4.2. *Results*

In this section, we report the results of our estimation of the effects of the minimum wage. Our preferred specification uses lognormality for the latent wage distribution and a linear in wage specification for the logistic regression of the conditional probability of formality given the wage. Further details about the estimation can be found in the Appendix D.

Although our model estimates allow us to obtain estimates of the effects of the minimum wage on the wage distribution, our main interest lies in the effects of the minimum wage on the size of the formal sector. The reasons for this are twofold: First, the effects of the minimum wage on the wage distribution depend heavily on our ability to correctly characterize the spillover effects of the policy, whereas the effects of the minimum wage on the size of the formal sector do not. [21] Second, the labor reallocation across sectors as a result of the policy is what allows us to obtain

[21] It can be shown that our estimates are invariant to misspecification of the effects of the minimum wage on the upper part of the wage distribution or the shape of the latent wage distribution itself whenever the conditional probability of latent sector given latent wage is flat. We find that in our application, this assumption is close to being satisfied.

information about the taxonomy of contract types we defined earlier in the paper. For those reasons, we focus mainly on the effects on the size of the formal sector, and we treat the other parameters such as the effects of the minimum wage on average wages – although surely of interest on their own– as a nuisance for the purposes of our exercise.

In examining the point estimates and standard errors in Table II, we observe sizable estimates of the unemployment effects of the minimum wage. The results also indicate that the minimum wage affects wages in both the formal and informal sectors. The evidence from Table II suggests that sector mobility is limited. The estimates of the sector-mobility parameter ($\pi_d^{(1)}$) are approximately 12%, averaging across different years.

We estimate the latent size of the formal sector of approximately 78% of the economy (taking the year 2004 as an example). This implies that the minimum wage reduces the size of the formal sector by approximately 8%. The informal sector, on the other hand, grows approximately by 27%, from about 22% to 28% of the economy. This larger effect in the relative size of the informal sector is explained by the fact that this sector is approximately 3.5 times smaller than the formal sector in the absence of the policy. These values are similar to, although smaller than, the results obtained by Jales (2018) using a closely related reduced-form framework.[22] Again taking the year 2004 as an example and using our welfare approximation formula, we find that the minimum wage increases the ex-ante expected welfare of formal workers affected

[22] Jales' (2018) model is more flexible in terms of the shape of the latent wage distribution, which is there assumed only to be continuous but more restrictive in the shape of the conditional probability of latent sector given the wage, which is assumed to be of order zero, or flat. This implies that, in the absence of the minimum wage, the wage distribution of formal and informal sectors would be indistinguishable from one another. This assumption is shown to be a useful approximation for the case of the Brazilian labor market, but we believe that the approach we take here yields a better approximation. In other words, the approximation error by assuming normality while being more flexible in terms of the conditional probability of sector given the wage is likely to be smaller than the error of assuming independence between latent sector and wages and continuous latent wage distribution. The framework we use here is also able to accommodate both spillover effects and richer structure for the effects of the minimum wage in the bottom part of the wage distribution. The results we obtain allowing for spillovers are discussed in Appendix D.

Table II: Parameter Estimates

Parameter		2001	2002	2003	2004	2005
Aggregate						
π_d	Non-compliance	0.20***	0.22***	0.20***	0.21***	0.19***
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
π_m	Bunching	0.24***	0.36***	0.28***	0.25***	0.37***
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
π_u	Non-employment	0.56***	0.42***	0.52***	0.54***	0.44***
		(0.01)	(0.02)	(0.01)	(0.01)	(0.02)
$Pr[W(0) < m]$	Fraction Affected	0.27***	0.27***	0.33***	0.33***	0.33***
		(0.00)	(0.00)	(0.00)	(0.00)	(0.01)
$Pr[S(0) = 1]$	Latent size of the formal sector	0.76***	0.78***	0.78***	0.78***	0.81***
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Formal Sector						
$\pi_d^{(1)}$	Sector mobility	0.04***	0.21***	0.09***	0.08***	0.19***
		(0.01)	(0.02)	(0.01)	(0.01)	(0.02)
$\pi_m^{(1)}$	Bunching	0.19***	0.23***	0.22***	0.20***	0.26***
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$\pi_u^{(1)}$	Non-employment	0.77***	0.56***	0.69***	0.71***	0.55***
		(0.02)	(0.02)	(0.02)	(0.01)	(0.02)
Informal Sector						
$\pi_d^{(0)}$	Non-compliance	0.62***	0.24***	0.56***	0.59***	0.21***
		(0.01)	(0.03)	(0.02)	(0.02)	(0.03)
$\pi_m^{(0)}$	Bunching	0.38***	0.76***	0.44***	0.41***	0.79***
		(0.01)	(0.03)	(0.02)	(0.02)	(0.03)

Note: Bootstrapped standard errors (computed using 100 replications) are given in parentheses.

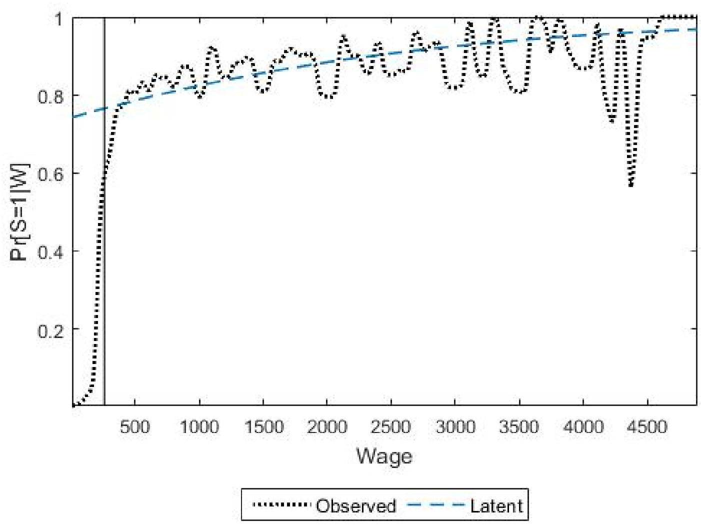
by the policy by around 52 Brazilian Reais. This is roughly 7% of the average wage earned by formal sector workers in that year.[23] Lastly, these estimates imply that – for the subset of formal worker-firm pairs that the minimum wage is binding – the fraction of contracts of the first kind must be between 71 and 91 percent, whereas the fraction of contracts of the second kind must be between 4 and 23 percent.[24]

Figure 7 displays the observed and latent conditional probability of formality with respect to wage. We can graphically observe the small elasticity of (latent) formality with respect to the wage around the minimum wage level from the apparent horizontal shape of this curve. This flatness contrasts with the steep slope of the *observed* conditional probability of the sector given the wage, which discontinuously jumps at

[23]We stress that our welfare approximation is only valid for small enough minimum wage levels and it does not include any spillover or general equilibrium effect of the policy, so this particular estimate is subject to a much higher degree of uncertainty than the other estimates we report.

[24]These bounds come from the fact that the type of contract is identified provided that the worker firm pair either moves to the informal sector or ends the employment relationship. For those that bunch at the minimum wage, the minimum wage does not increase the costs of formality to an extent large enough to identify the nature of the contract.

Figure 7: Conditional Probability of the Sector Given the Wage

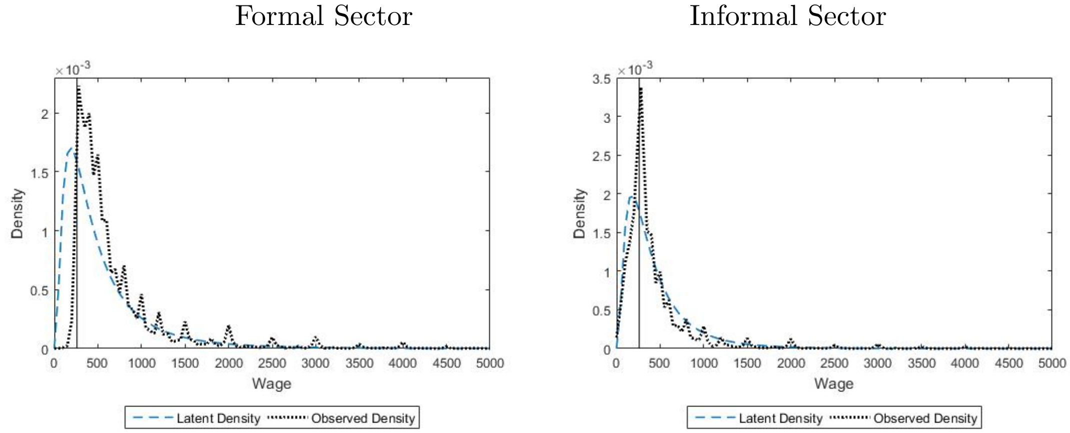


Note: Observed conditional probabilities based on a non-parametric local constant (Nadaraya-Watson) estimator using a gaussian kernel (bandwidth = R$30). Year 2004.

the minimum wage level.

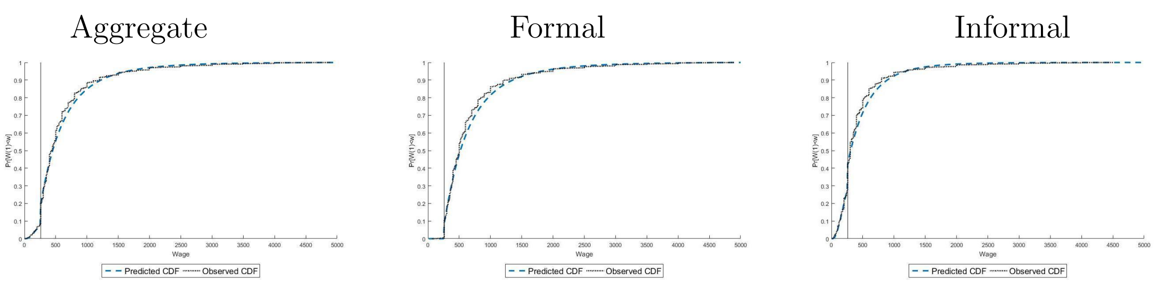
Figure 8 displays the observed and latent densities for the formal and informal sectors based on the model parameter estimates for the year 2004. The latent wage distribution tends to be below the observed distribution for the formal sector for values above the minimum wage. This is a consequence of workers moving away from the formal sector (into either unemployment or informal employment). The sector-mobility channel increases the measured density above the minimum wage due to a rescaling effect. The informal sector, as predicted by the model, behaves in the opposite way: The observed density tends to be below the latent density for values above the minimum wage. This result is due to the inflow of workers from the formal sector, which induces a rescaling of the density and reduces its values above the minimum wage. Figure 9 displays the fit of the model for the cumulative distribution functions of the unconditional and conditional wage distributions. The model seems to be able to capture the most important features of the joint distribution of sector and wage

Figure 8: Latent and Observed Densities



Note: Density estimates using a gaussian kernel (bandwidth = R$30). Year 2004.

Figure 9: Observed and Predicted CDFs



Note: Year 2004.

with a relatively small number of parameters. We discuss in greater detail the model fit in Appendix D. In the appendix, we also discuss the robustness of these estimates to the restrictions we impose on spillovers and also on the form through which the minimum wage affects the bottom part of the wage distribution.

4.2.1. *Marginal Effects of the Minimum Wage*

In this section, we compute the effects of changes in the minimum wage level, using a plug-in approach for the terms appearing the expressions for the marginal effects. More details are discussed in Appendix E.

Table IV displays the estimated effects of *changes* in the minimum wage level implied

Table IV: Marginal Effects

Parameter	Expression	2001	2002	2003	2004	2005
Average Wage						
Aggregate	$\frac{\partial E[W(1)]}{\partial m}$	8.40***	6.72***	8.41***	8.28***	7.18***
		(0.30)	(0.29)	(0.28)	(0.23)	(0.34)
Formal Sector	$\frac{\partial E[W(1)\|S(1)=1]}{\partial m}$	16.16***	12.60***	16.54***	15.97***	13.23***
		(0.56)	(0.55)	(0.53)	(0.45)	(0.64)
Informal Sector	$\frac{\partial E[W(1)\|S(1)=0]}{\partial m}$	1.61***	0.28	1.53***	1.44***	1.19***
		(0.21)	(0.20)	(0.18)	(0.16)	(0.16)
Employment						
Aggregate (% Change)	$\frac{\partial c}{\partial m}\frac{1}{c} \times 100$	-1.96***	-1.28***	-1.71***	-1.65***	-1.13***
		(0.08)	(0.08)	(0.08)	(0.06)	(0.09)
Formal Sector (% Change)	$\frac{\partial c^{(1)}}{\partial m}\frac{1}{c^{(1)}} \times 100$	-3.62***	-2.59***	-3.37***	-3.12***	-2.38***
		(0.14)	(0.13)	(0.12)	(0.10)	(0.14)
Informal Sector (% Change)	$\frac{\partial c^{(0)}}{\partial m}\frac{1}{c^{(0)}} \times 100$	0.31***	1.60***	0.74***	0.68***	1.41***
		(0.11)	(0.10)	(0.09)	(0.09)	(0.08)
Relative size (% Change)	$\frac{\partial \log(Pr[S(1)=1]/Pr[S(1)=0])}{\partial m} \times 100$	-3.93***	-4.18***	-4.11***	-3.81***	-3.79***
		(0.07)	(0.07)	(0.07)	(0.07)	(0.08)

Note: Marginal effect estimates multiplied by a typical change (R$20.00) in the minimum wage. Bootstrapped standard errors (computed using 100 replications) are given in parentheses.

by the estimates of the structural parameters. We computed the effects of the minimum wage on average wages, on employment, and on the relative size of the formal sector. To place the numbers in perspective, we multiply the marginal effects obtained by a typical change in the value of the minimum wage observed in the analyzed period (R$20).[25]

The estimates show that the minimum wage increases wages for the aggregate economy. The estimated effect is approximately R$7.80, or 40% of the change in the minimum wage. The estimated effects on average wages show a larger effect in the formal sector, of approximately R$14.90. Both of these effects are driven by the increase in the wage of low-wage workers *and* the decrease in the proportion of low-wage workers.

The results indicate an effect close to zero for wages in the informal sector. The small estimated effects of the minimum wage on average wages in the informal sector result from a combination of higher wages for some informal workers and the inflow of low-wage workers to the informal sector. This latter channel decreases the perceived effect

[25]Table IV reports the marginal effects of the minimum wage multiplied by a factor of 20. These estimates provide an approximation of the effect of a discrete change in the minimum wage.

of the policy in the informal sector.

Averaging the estimates across the period considered, the results suggest an approximately 1.6% decrease in employment following a typical (and exogenous) change in the minimum wage level. The decrease in the size of the formal sector is larger, approximately 3%. The estimates show that the informal sector experiences a 0.95% increase in employment. In terms of the relative size of the formal sector, the estimates suggest a decrease of approximately 3.97%. To place this number in perspective, the size of the formal sector in 2004 is 0.72. This estimate suggests that an exogenous increase (of R$20,00) in the minimum wage level would induce the formal sector to decrease to 0.71, that is, to decrease by one percentage point. This effect takes into account the outflow of workers from the formal sector, the inflow of workers to the informal sector, and the decrease in the size of the formal sector due to unemployment.[26]

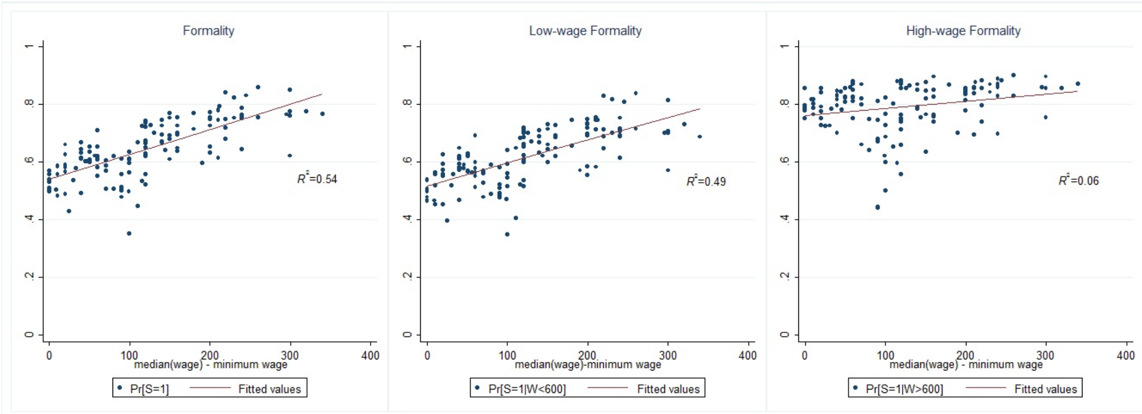
4.3. *Reduced-form evidence*

Figure 10 displays levels of formality across states by year, as a function of a standard measure of the strength of the minimum wage: the distance between the median wage and the minimum wage level. If the minimum wage decreases the size of the formal sector, one should expect, *ceteris paribus*, that comparable states with a larger fraction of workers affected by the policy to present smaller levels of formality. One reasonable concern, though, is that these states might not be comparable: States that are different in terms of their wage distributions are also different in terms of their latent levels of formality. We can, however, gather some evidence of these differences by looking at points in the wage distribution in which we do not expect the minimum wage to have any substantive effect on formality.

The graph on the left corner of Figure 10 displays the aggregate level of formality at each state by year, as a function of the distance between the median wage and the

[26]This exercise highlights that the effect of marginal changes in the minimum wage may be too small to be detected using time-series variation, especially if minimum wage changes are more likely to happen during the boom part of business cycles. Nevertheless, the cumulative effect of all minimum wage changes on the size of the formal sector may not be inconsequential.

Figure 10: Formality and wages across states by year



minimum wage. We see there a strong relationship: States for which the minimum wage is more binding tend to present smaller levels of formality. This relationship may or may not be related to the causal effect of the minimum wage since these differences could be attributed to general differences in the level of formality across states that happen to correlate with the minimum wage strength. The third graph, however, suggests that this should not be a serious concern. Once one looks at the levels of formality for wages above R\$600, the relationship between the minimum wage and the level of formality becomes weak. For wages above R\$600,00, we do not expect any difference to be attributed to minimum wage effects since this threshold rules out workers that are directly affected by the policy. Thus, the relationship observed in the right graph is a measure of how comparable the levels of formality are across different states, at least for the workers with wages higher than R\$600,00. The results suggest that, although far from being a perfect control group, the bias associated with latent differences in formality levels across states should be fairly small. This is certainly true for the upper part of the wage distribution. If one believes that this is also true for the bottom part of the wage distribution, then looking at the levels of formality for low-wage groups could be informative about the effects of the minimum wage on the size of the formal sector.

The center graph in Figure 10 displays the relationship between the minimum wage and the level of formality for the bottom part of the wage distribution. In contrast with the graph on the right, there seems to be a strong relationship between how close the minimum wage is to the median wage and how small the size of the formal sector for low-wage workers is. This also shows that the differences between the levels of formality across states are driven mainly by differences in the levels of formality that prevail *at the bottom* of their wage distributions. The minimum wage stands out as the most immediate explanation for this empirical regularity.

This evidence is in line with the main results of our empirical exercise: The minimum wage seems to cause a decrease in the size of the formal sector. Once one compares states by how strongly they were affected by the minimum wage policy, these states tend to have a substantially smaller size of the formal sector for the bottom part of the wage distribution, even though they seem to have somewhat comparable levels of the size of the formal sector in the upper part of their wage distributions.

5. CONCLUSION

This paper uses a bargaining framework, in which workers and firms engage in a bargain over the wage and the sector in which production will take place, to rationalize the joint distribution of employment, sector, and wages. This framework is shown to be useful to understand the effects of policies that increase the costs of operating in the informal sector (such as increased enforcement) and policies that impose constraints on firms operating in the formal sector, such as the minimum wage. An interesting feature of this bargaining framework is that differences in wages across sectors will, in general, reflect both productivity differences – as in Roy (1951) – as well as compensating differentials – as in Rosen (1986)–. In the limiting cases in which either the work or the firm holds all the bargaining power, then wages start to reflect only one of these distinct economic forces.

In our empirical exercise, we evaluate the effects of the minimum wage in the Brazilian economy using a dual economy statistical model under a parametric assumption regarding the shape of the latent wage distribution. The model seems to approximate most of the stylized facts concerning the joint distribution of sectors and wages using a small number of parameters.

We find that the minimum wage generates statistically significant non-employment effects. The policy also leads a fraction – approximately 6% – of the affected workers to move from the formal to the informal sector of the economy. This sector-mobility channel, combined with the non-employment effects, leads to a decrease of approximately 10% in the size of the formal sector when compared to the counterfactual scenario of the absence of the minimum wage. Small sector-mobility probabilities induce large changes in the relative size of the informal sector since the latent size of the formal sector is approximately 3.5 times larger than the informal sector. Our estimates suggest that the informal sector experiences a growth of around 27%, from 22% to 28% of the economy, as a result of the minimum wage policy.

A decomposition exercise based on the estimates of the model parameters suggests that unemployment effects and the inflow of low-wage workers to the informal sector are responsible for most of the observed differences in the wage distribution across sectors. Reduced-form evidence of such a sector mobility channel is in line with the model's prediction, that is, the differences in the size of the formal sector across states for low- and high-wage groups replicate the patterns predicted by the model. These results highlight the importance of accounting for the relationship between the level of formality and labor market policies, such as the minimum wage, when analyzing the effects of these policies in developing countries.

REFERENCES

Mariano Bosch, Edwin Goni, and William F Maloney. The determinants of rising informality in Brazil: Evidence from gross worker flows. *World Bank Policy Research Working Paper Series, Vol*, 2007.

Pierre Brochu and David A Green. The impact of minimum wages on labour market transitions. *The Economic Journal*, 123(573):1203–1235, 2013.

Charles Brown. Equalizing differences in the labor market. *The Quarterly Journal of Economics*, 94(1):113–134, 1980.

Sebastian Calonico, Matias D Cattaneo, and Rocio Titiunik. Robust nonparametric confidence intervals for regression-discontinuity designs. *Econometrica*, 82(6):2295–2326, 2014.

Doruk Cengiz, Arindrajit Dube, Attila Lindner, and Ben Zipperer. The effect of minimum wages on low-wage jobs. *The Quarterly Journal of Economics*, 134(3):1405–1454, 2019.

Hernando De Soto. Theother path: The invisible revolution in the third world, 1989.

Joseph J Doyle Jr. Employment effects of a minimum wage: A density discontinuity design revisited.

Niklas Engbom and Christian Moser. Earnings inequality and the minimum wage: Evidence from brazil. 2017.

Christopher J Flinn. Minimum wage effects on labor market outcomes under search, matching, and endogenous contact rates. *Econometrica*, 74(4):1013–1062, 2006.

Miguel N Foguel and Carlos Henrique Corseuil. What drives the effect of labor inspections on formal labor? the roles of information and punishment.

Sebastian Galiani and Federico Weinschelbaum. Modeling informality formally: households and firms. *Economic Inquiry*, 50(3):821–838, 2012.

Wilson Guzman. The effects of increased enforcement of the national minimum wage law in a developing country: The case of ecuador. 2017.

Péter Harasztosi and Attila Lindner. Who pays for the minimum wage? *American Economic Review*, 109(8):2693–2727, 2019.

James Heckman and Guilherme Sedlacek. The impact of the minimum wage on the employment and earnings of workers in South Carolina. *Report of the Minimum Wage Study Commission*, 5: 225–272, 1981.

Hugo Jales and Zhengfei Yu. Identification and estimation using a density discontinuity approach. In *Regression Discontinuity Designs: Theory and Applications*, pages 29–72. Emerald Publishing Limited, 2017.

Henrik J Kleven and Mazhar Waseem. Using notches to uncover optimization frictions and structural elasticities: Theory and evidence from Pakistan. *The Quarterly Journal of Economics*, 128(2): 669–723, 2013.

Rafael La Porta and Andrei Shleifer. The unofficial economy and economic development. *Brookings Papers on Economic Activity*, 2008.

Rafael La Porta and Andrei Shleifer. Informality and development. *The Journal of Economic Perspectives*, 28(3):109–126, 2014.

Guy Laroque and Bernard Salanié. Labour market institutions and employment in france. *Journal of Applied Econometrics*, 17(1):25–48, 2002.

Adam Lavecchia. Minimum wage policy with optimal taxes and unemployment. *McMaster Univer-*

sity Department of Economics Working Paper, (2019-03), 2019.

David Lee and Emmanuel Saez. Optimal minimum wage policy in competitive labor markets. *Journal of Public Economics*, 96(9):739–749, 2012.

Enlinson Mattos and Laudo M Ogura. Skill differentiation between formal and informal employment. *Journal of Economic Studies*, 36(5):461–480, 2009.

Costas Meghir, Renata Narita, and Jean-Marc Robin. Wages and informality in developing countries. *The American Economic Review*, 105(4):1509–1546, 2015.

R H Meyer and D A Wise. Discontinuous Distributions and Missing Persons: The Minimum Wage and Unemployed Youth. *Econometrica*, 51, Issue:1677–1698, 1983.

Joan Monras. Minimum wages and spatial equilibrium: Theory and evidence. *Journal of Labor Economics*, 37(3):853–904, 2019.

Andrea Otero-Cortes. *Essays on Labor Informality in Developing Countries.* PhD thesis, The University of North Carolina at Chapel Hill, 2018.

James E Rauch. Modelling the informal sector formally. *Journal of development Economics*, 35(1): 33–47, 1991.

Sherwin Rosen. The theory of equalizing differences. *Handbook of labor economics*, 1:641–692, 1986.

Andrew Donald Roy. The distribution of earnings and of individual output. *The Economic Journal*, 60(239):489–505, 1950.

Andrew Donald Roy. Some thoughts on the distribution of earnings. *Oxford economic papers*, 3(2): 135–146, 1951.

Emmanuel Saez. Do taxpayers bunch at kink points? *American Economic Journal: Economic Policy*, 2(3):180–212, 2010.

Fernando Saltiel and Sergio Urzúa. Does an increasing minimum wage reduce formal sector employment? evidence from brazil. 2018.

David Pence Slichter. The employment effects of the minimum wage: A selection ratio approach to measuring treatment effects. 2015.

Isaac Sorkin. Are there long-run effects of the minimum wage? *Review of economic dynamics*, 18 (2):306–333, 2015.

Christopher Taber and Rune Vejlin. Estimation of a roy/search/compensating differential model of the labor market. 2016.

Coen N Teulings. Aggregation bias in elasticities of substitution and the minimum wage paradox. *International Economic Review*, 41(2):359–398, 2000.

James Tobin. Estimation of relationships for limited dependent variables. *Econometrica: journal of the Econometric Society*, pages 24–36, 1958.

Mirco Tonin. Minimum wage and tax evasion: Theory and evidence. *Journal of Public Economics*, 95(11-12):1635–1651, 2011.

Gabriel Ulyssea. Regulation of entry, labor market institutions and the informal sector. *Journal of Development Economics*, 91(1):87–99, 2010.

Gabriel Ulyssea. Firms, informality, and development: Theory and evidence from brazil. *American Economic Review*, 108(8):2015–47, 2018.

SUPPLEMENTAL APPENDIX – ON THE EFFECTS OF THE MINIMUM WAGE ON EMPLOYMENT, FORMALITY, AND THE WAGE DISTRIBUTION

HUGO JALES [1] AND ZHENGFEI YU [2]

CONTENTS

[1]Syracuse University, department of economics. Email: hbjales@syr.edu.
[2]University of Tsukuba, Faculty of Humanities and Social Sciences.

1. APPENDIX A: MODEL ANALYSIS

In this section, we first prove that (s^*, w^*) characterized by (2) solves the Nash bargaining problem (1) in the paper.

Recall that the worker's utility is given by $U(l, s, w,) = l \times (w - \epsilon + \eta_s)$ and the firm's profit is given by $\Pi(l, s, w) = l \times (\alpha - w - \tau_s)$. The equilibrium employment, sector, and wage is obtained by maximizing the Nash Product:

$$(l^*, s^*, s^*) = \arg\max U(l, s, w)^{\nu}\Pi(l, s, w)^{1-\nu}, \tag{A.1}$$

where ν is the worker's bargaining power and $1 - \nu$ is the firm's bargaining power. A simple way to find the maximum of the Nash product in this case is to first set s to one, and then maximize over w. Denote the Nash Product when evaluated at $l = 1$, $s = 1$ – so the worker is employed and the constract is formal – at the wage w_1 that maximizes the Nash product given $l = 1$ and $s = 1$ by $B(1, 1, w_1)$. Denote the Nash Product when $l = 1$ and $s = 0$ – so the worker is employed and the contract is informal – at the wage w_0 that maximizes the Nash Product given $l = 1$ and $s = 0$ by $B(1, 0, w_0)$, Lastly, compare the maximum obtained when s is set to one, the maximum obtained when $s = 0$ – that is, compare the values of $B(1, 1, w_1)$ and $B(1, 0, w_0)$. Given l and s, the w_s that maximizes the Nash Product can be found by taking the first order condition of $B(l, s, w)$ with respect to w. This yields:

$$w_s^* = \nu(\alpha - \tau_s) + (1 - \nu)(\epsilon - \eta_s). \tag{A.2}$$

when $\nu = 1/2$ this collapses to the wage equations we use in the simplified version of the model presented on the paper, so that $w_1 = \frac{1}{2}(\alpha + \epsilon - \eta_1 - \tau_1)$ and $w_0 = \frac{1}{2}(\alpha + \epsilon - \eta_0 - \tau_0)$. Evaluating $\Pi(l = 1, s, w)$ and $U(l = 1, s, w)$ at the optimal value of $w = w_s$, we obtain $U(1, s, w_s) = \nu r_s$ and $\Pi(1, s, w_s) = (1-\nu)r_s$, where $r_s = \alpha - \epsilon + \eta_s - \tau_s$ is the surplus of the match when the worker is assigned to sector s. Using these expressions, we can obtain the value of the Nash Products when the worker is assigned to each sector: When $s = 1$, we have that $B(1, 1, w_1) = \upsilon^{\nu}(1 - \nu)^{1-\nu} r_1$, and when $s = 0$, we have that $B(1, 0, w_0) = \upsilon^{\nu}(1 - \nu)^{1-\nu} r_0$. This implies that $B(1, 1, w_1)$ is greater than $B(1, 0, w_0)$ only if $r_1 > r_0$. This yields the optimal sector assignment for all workers that employed workers: $s = \mathbb{1}\{r_1 > r_0\}$. Plugging in the definitions of the formal and informal sectors' match surpluses and removing the common terms, we obtain:

(A.3)

$$s^* = \mathbb{1}\{r_1 > r_0\} = \mathbb{1}\{\alpha - \epsilon + \eta_1 - \tau_1 > \alpha - \epsilon + \eta_0 - \tau_0\} = \mathbb{1}\{\eta_1 - \eta_0 > \tau_1 - \tau_0\},$$

which proves the solution (2) in the paper.

In the following, we prove Propositions 1 and 2. Recall that both propositions have three parts. We focus on the proof of proposition 1 since the proof of proposition 2 is analogous. We start with some definitions. The compensating wage differential is the difference in the workers utility across two distinct sector allocations, when evaluated at the same wage. It measures the worker's willingness to pay for the job the (relative) amenities of the job. In our setting, the compensating wage differentials is then defined as $U(1, 1, w) - U(1, 0, w)$. Evaluating this expression, we find that the compensating wage differential is given by the difference in job amenities $\eta_1 - \eta_0$. Similarly, the sector productivity differential is the difference between the firm's profit when hiring the worker in the formal sector when compared to hiring the worker in a informal contract, when both of these options are evaluated at the same wage. It measures the differences in the worker's productivity (net of taxes, sector-specific costs and distortions) when he is assigned to different types of employment. In our setting, the sector productivity differential is defined as $\Pi(1, 1, w) - \Pi(1, 0, w)$. Evaluating this expression in the context of our model, we find that the sector productivity differential is given by $\tau_1 - \tau_0$. Lastly, we say that a joint distribution of sector and wages is consistent with the (standard) Roy model if the joint distribution of (w_1, w_0, s) is such that $s = 1$ if, and only if, $w_1 > w_0$. That is, sector choice is based on income maximization, and income maximization alone.[1]

Proof of Proposition 1: Recall from the discussion of the previous section that, for any bargaining power ν, the wage equation is given by $w_s = \nu(\alpha - \tau_s) + (1-\nu)(\epsilon - \eta_s)$. Taking $\nu = 1$ yields the particular case in which the wage equation reduces to $w_s = \alpha - \tau_s$. This establishes the first claim of part (a) of proposition one. Taking partial derivatives of w_s with respect to τ_s, α, and η_s, yields the second result: $\frac{\partial w_s}{\partial \alpha} = 1$, $\frac{\partial w_s}{\partial \tau_s} = -1$, and $\frac{\partial w_s}{\partial \eta_s} = 0$. This yields the second claim. Finally, taking the difference between w_1 and w_0 yields the conclusion that $\Pi(1,1,w) - \Pi(1,0,w)$ is equal to $w_1 - w_0$, so the differences in wages across different sector assignments are the differences in the worker's productivity across sectors. This completes the proof of the last result in part (a) of the proposition.

To prove part (b), recall that, for any bargaining power, we have that $s = \mathbb{1}\{\eta_1 - \eta_0 > \tau_1 - \tau_0\}$. Then, if $\eta_1 = \eta_0$, then the sector choice equation collapses to $s = \mathbb{1}\{\tau_1 < \tau_0\}$, which in this case, using the simplified wage equation $w_s = \alpha - \tau_s$, leads us to conclude that $s = \mathbb{1}\{w_1 > w_0\}$. This finally yields the conclusion that when $\eta_1 = \eta_0$ and $\nu = 1$, the joint distribution of (w_1, w_0, s) is indistinguishable from the standard Roy model.

To prove part (c), we assume instead that $\tau_1 - \tau_0$ is constant across workers and that $Cov(\alpha, s^*) = 0$. Then, $\mathbb{E}[w_1|s=1] - \mathbb{E}[w_0|s=0] = \mathbb{E}[\alpha - \tau_1|s=1] - \mathbb{E}[\alpha - \tau_0|s=0] = \tau_1 - \tau_0 = \Pi(1,1,w) - \Pi(1.0,w)$, where the first equality follows from the wage equation, the second follows from the assumption on the covariance between α and s, and the third follows from the assumption on the heterogeneity in τ across workers. In this case, differences in wages across sectors identify sector productivity differentials. Taking $\nu = 0$ yields the particular case in which the wage equation reduces to $w_s = \epsilon - \eta_s$. Taking the difference between w_1 and w_0 yields the conclusion that $U(1,1,w) - U(1,0,w)$ is equal to $w_1 - w_0$, so the differences in wages across different sector

[1]The Roy model is typically further parametrized so the joint distribution of (w_1, w_0) is assumed to be bivariate normal, but this is not as important as the relationship between sector choice and income maximization.

assignments are compensating wage differentials. Now, if $\tau_1 = \tau_0$, then the sector choice equation collapses to $s = \mathbb{I}\{\eta_1 > \eta_0\}$, which in this case is identical to $s = \mathbb{I}\{w_1 < w_0\}$, which yields the conclusion that when $\eta_1 = \eta_0$ and $\nu = 1$, the joint distribution of (w_1, w_0, s) is indistinguishable from a Roy model featuring seemingly irrational selection, that is, $s = \mathbb{I}\{w_1 < w_0\}$. Lastly, instead, assume that $\eta_1 - \eta_0$ is constant across workers and $Cov(\epsilon, s^*) = 0$. Then, $\mathbb{E}[w_1|s = 1] - \mathbb{E}[w_0|s = 0] = \mathbb{E}[\epsilon - \eta_1|s = 1] - \mathbb{E}[\epsilon - \eta_0|s = 0] = \tau_1 - \tau_0 = U(1, 1, w) - U(1, 0, w)$, where the first equality follows from the wage equation, the second follows from the assumption on the covariance between ϵ and s^* and the third follows from the assumption on the heterogeneity in η across workers. In this case, differences in wages across sectors identify compensating wage differentials.

2. APPENDIX B: WELFARE

2.1. *Welfare Formulas*

From the structure imposed by the model presented in Section 2, it is possible to characterize the welfare effects of the minimum wage. Here we derive the expressions for both worker's welfare and also aggregate welfare,[2] defined as the sum of workers' utility and firm's profits. Recall that $w_s = \frac{1}{2}(\alpha + \epsilon - \eta_s - \tau_s)$ let and $\Delta = \frac{1}{2}(\eta_1 - \eta_0 - \tau_1 + \tau_0)$. Workers are employed formally whenever $\Delta > 0$. Define the aggregate welfare:

$$r^*_{s^*} \equiv \Pi(l^*, s^*, w^*) + U(l^*, s^*, w^*) \tag{A.4}$$

In the presence of the minimum wage, aggregate welfare will be given by the sum of profits and utility that prevail at the equilibrium allocation under the effects of the policy:

$$\tilde{r}_{\tilde{s}} \equiv \Pi(\tilde{l}, \tilde{s}, \tilde{w}) + U(\tilde{l}, \tilde{s}, \tilde{w}) \tag{A.5}$$

[2]Note that we do not include the tax revenues in the aggregate welfare definition.

The difference between (A.5) and (A.4) corresponds to the welfare effect of the minimum wage. The expected value of that difference writes:

$$\begin{aligned}\mathbb{E}[\tilde{r}_{\tilde{s}} - r^*_{s^*}] &= -\mathbb{E}[r^*_{s^*}|w^* < m, \tilde{l} = 0]\Pr[w^* < m, l^* = 0] \\ &\quad -\mathbb{E}\left[(\eta_1 - \tau_1) - (\eta_0 - \tau_0)|w^* < m, s^* > \tilde{s}\right]\Pr[w^* < m, s^* > \tilde{s}].\end{aligned}$$

Regarding the expression above, note that the welfare changes only for workers for which the minimum wage "bites' –that is – workers for which $w^* < m$. Also, when the worker-firm pair respond to the policy by setting the wage equal to m while keeping the worker in the same sector, then the welfare effect of this response is zero. This is so because, in this event, the minimum wage acts simply as a transfer of surplus from the firm to the worker. However, if the response is to destroy the match, then the welfare effect of the policy in this event is the negative of the surplus associated with that match. Lastly, when the worker moves to the informal sector, then the welfare effect of this change is given by minus $(\eta_1 - \tau_1) - (\eta_0 - \tau_0)$. This term is negative since the policy is inducing the worker to be assigned to a sub-optimal sector given his draw of η_s and τ_s. It is, thus, straightforward to see that the aggregate welfare effects of the minimum wage in this setting are negative.

Finally, it is interesting to note that the Bargain framework something that looks like a version of "efficient rationing" in the sense of Lee and Saez (2012).[3] It is possible to characterize the reactions of worker-firm pairs according to a set of inequalities associated with the difference between the worker's productivity and his outside option. If the worker moves to the informal sector, we have the following inequalities $\tau_1 - \eta_1 \leq \tau_0 - \eta_0 \leq \alpha - \epsilon$. This means that the surplus when formal is higher than the surplus when informal. However, the surplus when informal is still positive. If the worker-firm pair decides to end the match, we have the following inequalities $\tau_1 - \eta_1 \leq \alpha - \epsilon \leq \tau_0 - \eta_0$. This means that the difference between the worker's productivity and its outside option has to be bounded from above by the difference between the costs of informality and the worker's valuation of the amenities in the informal

[3]We thank Adam Lavecchia for pointing out this connection to us.

sector. Furthermore, if a worker loses the job, it must be the case that $\alpha \leq m + \tau_1$. These two inequalities limit the negative welfare effect of the minimum wage since the unemployment falls on matches with productivity α bounded from above by $m + \tau_1$ and also by $\epsilon + \tau_0 - \eta_0$. We can obtain a similar expression for the welfare of workers:

$$\begin{aligned}\mathbb{E}[U(l^*, s^*, w^*) - U(\tilde{l}, \tilde{s}, \tilde{w})] &= -\mathbb{E}[U(l^*, s^*, w^*)|\tilde{l} = 0, w^* < m] \Pr[\tilde{l} = 0, w^* < m] \\ &\quad + \mathbb{E}[m - w^*|\tilde{w} = m] \Pr[\tilde{w} = m] \\ &\quad + \frac{1}{2}\mathbb{E}[(\eta_1 - \tau_1) - (\eta_0 - \tau_0)|\tilde{s} = 1, s^* = 0] \Pr[\tilde{s} = 1, s^* = 0].\end{aligned}$$

This expression states that the expected change on worker's welfare by the minimum wage policy is given by an weighted average of the loss of utility associated with the disemployment effect, the gain associated with the wage increases, and the losses associated with inducing workers to move to the informal sector, where each of these factors are weighted by the probability of these events. Differently than in the case of aggregate welfare, the effects of the minimum wage on worker's welfare are not guaranteed to be negative. The welfare of workers can increase if the term associated with the workers that bunch at the minimum wage is large enough.

2.2. *The effect of a "small" minimum wage*

Let $r_s = \frac{1}{2}(\alpha - \tau_s - \epsilon + \eta_s)$ for $s = 1, 2$. Note that r_1 and r_0 are the surplus for the worker and the firm in the formal and informal sector respectively. The Nash Product in the formal sector with $w = m$ is

$$\begin{aligned}B(1,1,m) &= [(m - \epsilon + \eta_1)(\alpha - m - \tau_1)]^{1/2} \\ &= \left[\frac{1}{4}(\alpha - \tau_1 - \epsilon + \eta_1)^2 - \left(m - \frac{1}{2}(\alpha + \epsilon - \tau_1 - \eta_1)\right)^2\right]^{1/2} \\ &= \sqrt{r_1^2 - (m - w_1)^2}.\end{aligned}$$

On the other hand, the Nash Product for the informal sector is $B(1, 0, w_0) = r_0$. Recall that $\Delta = (\eta_1 - \eta_0) - (\tau_1 - \tau_0) = r_1 - r_0$.

Proof of Proposition 3: To simplify notations, we use "shift", "bunch" and "unemp" to denote consequences of minimum wage: workers shifting from formal to informal sector, bunching at the minimum wage and becoming unemployment. Note that these three events correspond to the conditioning events in the statement of Proposition 3.

(a). Workers shifting from formal to informal sector due to the minimum wage are characterized by $\Delta \geq 0$, $w_1 < m$, $r_1^2 - (m - w_1)^2 < r_0^2$, and $r_0 > 0$.

For arbitrary small value $c > 0$, if the minimum wage $m \leq c^2$, then

$$\begin{aligned}
\Pr\left[\Delta \leq c | shift\right] &= \frac{\Pr\left[0 \leq r_1 - r_0 \leq c, r_1^2 - (m - w_1)^2 < r_0^2, w_1 < m, r_0 > 0\right]}{\Pr\left[r_1 - r_0 \geq 0, r_1^2 - (m - w_1)^2 < r_0^2, w_1 < m, r_0 > 0\right]} \\
&= \frac{\Pr\left[r_1 - r_0 \geq 0, r_1^2 - (m - w_1)^2 < r_0^2, w_1 < m, r_0 > 0\right]}{\Pr\left[r_1 - r_0 \geq 0, r_1^2 - (m - w_1)^2 < r_0^2, w_1 < m, r_0 > 0\right]} \\
&= 1,
\end{aligned}$$

where the second equality comes from that $r_1^2 - r_0^2 < (m - w_1)^2 \leq c^2 \Rightarrow r_1 - r_0 \leq c$.

(b). Workers bunching at the minimum wage m are characterized by $\Delta \geq 0$, $w_1 < m$, $r_1^2 - (m - w_1)^2 \geq r_0^2 \geq 0$, and $r_0 \geq 0$. Then for all m, including small m, we have,

$$\begin{aligned}
\lim_{c \downarrow 0} \Pr\left[\Delta \leq c | bunch\right] &= \lim_{\epsilon \downarrow 0} \frac{\Pr\left[0 \leq r_1 - r_0 \leq c, r_1^2 - (m - w_1)^2 \geq r_0^2 > 0, w_1 < m\right]}{\Pr\left[r_1 - r_0 \geq 0, r_1^2 - (m - w_1)^2 \geq r_0^2 > 0, w_1 < m\right]} \\
&= 0.
\end{aligned}$$

(c). Workers become unemployed if and only if $r_1 > 0$, $w_1 < m$, $r_1 < m - w_1$, and $r_0 \leq 0$. Then for all m, including small m, we have

$$\begin{aligned}
\lim_{c \downarrow 0} \Pr\left[\Delta \leq c | unemp\right] &= \lim_{c \downarrow 0} \frac{\Pr\left[r_1 - r_0 \leq c, r_1 < m - w_1, w_1 < m, r_1 > 0, r_0 \leq 0\right]}{\Pr\left[r_1 < m - w_1, w_1 < m, r_1 > 0, r_0 \leq 0\right]} \\
&= \lim_{c \downarrow 0} \frac{\Pr\left[0 < r_1 - r_0 \leq c, r_1 < m - w_1, w_1 < m, r_1 > 0, r_0 \leq 0\right]}{\Pr\left[r_1 < m - w_1, w_1 < m, r_1 > 0, r_0 \leq 0\right]} \\
&= 0.
\end{aligned}$$

(d). For any small value of c, if $m \leq c$, then $r_1 < m - w_1$ implies $r_1 \leq c$. Therefore, for $m \leq c$, we have

$$\begin{aligned}
\Pr\left[r_1 \leq c | unemp\right] &= \frac{\Pr\left[r_1 \leq c, r_1 < m - w_1, w_1 < m, r_1 > 0, r_0 \leq 0\right]}{\Pr\left[r_1 < m - w_1, w_1 < m, r_1 > 0, r_0 \leq 0\right]} \\
&= \frac{\Pr\left[r_1 < m - w_1, w_1 < m, r_1 > 0, r_0 \leq 0\right]}{\Pr\left[r_1 < m - w_1, w_1 < m, r_1 > 0, r_0 \leq 0\right]} \\
&= 1.
\end{aligned}$$

3. APPENDIX C: DISTRIBUTIONAL APPROXIMATIONS

In this section, we derive the results concerning the functional form for the joint distribution of sector and wages that prevails in this economy both in the presence and also in the absence of the minimum wage. These results allow us to arrive at a tractable structure that can be used to estimate the model parameters. Here we prove the results discussed in Section 3.1 of the paper.

3.1. *Latent wage distributions*

For $s \in \{0, 1\}$, consider $w_s = \frac{1}{2}(\alpha + \epsilon - \eta_s - \tau_s)$. Decompose $\epsilon - \eta_s - \tau_s$ into two parts: a part predicted by α and the remaining part.

$$\epsilon - \eta_s - \tau_s = \beta_{0s} + \beta_{1s}\alpha + u_s,$$

where β_{0s} and β_{1s} are constants and $\mathbb{E}[\alpha u_s] = \mathbb{E}[u_s] = 0$. Hence

$$w_s = \frac{1}{2}(\alpha + \beta_{0s} + \beta_{1s}\alpha + u_s) = \frac{1}{2}\alpha(1 + \beta_{1s})\left(1 + \frac{u_s}{\alpha(1 + \beta_{1s})}\right) + \frac{1}{2}\beta_{0s}.$$

We restate Assumptions 1 to 3 in more specific forms as follows.

ASSUMPTION A1 *$\log \alpha$ has a normal distribution $N(\mu, \sigma^2)$.*

ASSUMPTION A2 *There are small positive numbers t and κ such that $\Pr\left[\left|\frac{u_s}{\alpha(1+\beta_{1s})}\right| > t\right] \leq \kappa$.*

ASSUMPTION A3 *In the absence of the minimum wage, the support of w_s is $\mathbb{R}^+$ for $s \in \{1, 0\}$.*

Proof of Proposition 5: Proposition 5 immediately follows from Proposition A1 below. Note that $S(0) = \mathbb{1}\{\Delta > 0\}$.

PROPOSITION A1 *(i). Under Assumptions A1 to A3, w_s approximately has a log-normal distribution. That is, $\log w_s$ has a normal distribution with mean $\mu -$*

$\log((1+\beta_{1s})/2)$ *and standard error* σ.

(ii). Further assume that $(\log w_s, \Delta)'$ *approximately have a joint normal distribution with mean* $(\mu_s, \mu_\delta)'$ *and covariance* $\begin{bmatrix} \sigma^2 & \rho_s \sigma \sigma_\delta \\ \rho_s \sigma \sigma_\Delta & \sigma_\Delta^2 \end{bmatrix}$. *Then the conditional log wage densities* $f_{\log w_1}(x|\Delta \geq 0)$ *and* $f_{\log w_0}(x|\Delta < 0)$ *approximately have the following forms:*

$$f_{\log w_1}(x|\Delta \geq 0) \approx \frac{1}{\sigma}\phi\left(\frac{x-\mu_1}{\sigma}\right)\Phi\left(\frac{\frac{\mu_\Delta}{\sigma_\Delta} + \rho_1 \frac{x-\mu_1}{\sigma}}{\sqrt{1-\rho_1^2}}\right)/\Phi\left(\frac{\mu_\Delta}{\sigma_\Delta}\right), \tag{A.6}$$

$$f_{\log w_0}(x|\Delta < 0) \approx \frac{1}{\sigma}\phi\left(\frac{x-\mu_0}{\sigma}\right)\Phi\left(\frac{-\frac{\mu_\Delta}{\sigma_\Delta} - \rho_0 \frac{x-\mu_0}{\sigma}}{\sqrt{1-\rho_0^2}}\right)/\Phi\left(-\frac{\mu_\Delta}{\sigma_\Delta}\right). \tag{A.7}$$

(iii). The density of the latent wage w *is approximately*

$$f_{\log w}(x) \approx \frac{1}{\sigma}\left[\phi\left(\frac{x-\mu_1}{\sigma}\right)\Phi\left(\frac{\frac{\mu_\Delta}{\sigma_\Delta} + \rho_1 \frac{x-\mu_1}{\sigma}}{\sqrt{1-\rho_1^2}}\right) + \phi\left(\frac{x-\mu_0}{\sigma}\right)\Phi\left(\frac{-\frac{\mu_\Delta}{\sigma_\Delta} - \rho_0 \frac{x-\mu_0}{\sigma}}{\sqrt{1-\rho_0^2}}\right)\right].$$

PROOF: (i).

$$\Pr(w_s \leq x) = \Pr\left(\log\alpha + \log((1+\beta_{1s})/2) + \log\left(1 + \frac{u_s}{\alpha(1+\beta_{1s})}\right) \leq \log(x - \beta_{0s}/2)\right)$$

By Assumption A2,

$$\Pr\left(\log\left(1 + \frac{u_s}{\alpha(1+\beta_{1s})}\right) > \log(1+t)\right) \leq \kappa.$$

Since t is a small number, $\log(1+t) \approx t$. Then there is a small number $t' > t$ such that

$$\Pr\left(\log\left(1 + \frac{u_s}{\alpha(1+\beta_{1s})}\right) > t'\right) \leq \kappa, \text{ and } \Pr\left(\log\left(1 + \frac{u_s}{\alpha(1+\beta_{1s})}\right) < -t'\right) \leq \kappa.$$

Applying Lemma A1 (to be stated below) with $X = \log\alpha + \log((1+\beta_{1s})/2)$ and $Y = \log\left(1 + \frac{u_s}{\alpha(1+\beta_{1s})}\right)$, we have

$$\Pr(w_s \leq x) \approx \Pr(\log\alpha + \log((1+\beta_{1s})/2) \leq \log(x - \beta_{0s}/2)). \tag{A.8}$$

By Assumption A1, $\log\alpha + \log((1+\beta_{1s})/2)$ has a normal distribution with mean

$\mu - \log((1+\beta_{1s})/2)$ and standard error σ.

The support of w_s is $(\beta_{0s}/2, +\infty)$. By Assumption A3, $\beta_{0s} = 0$. Therefore, $\log w_s$ has a normal distribution with mean $\mu - \log((1+\beta_{1s})/2)$ and standard error σ.

(ii). We focus on $f_{\log w_1}(x|\Delta > 0)$. $f_{\log w_0}(x|\Delta < 0)$ can be shown in the same way.

$$\begin{aligned} f_{\log w_1}(x|\Delta > 0) &= \int_0^\infty f_{\log w_1, \Delta}(x, \delta) d\delta / \Pr(\Delta > 0) \\ &= f_{\log w_1}(x) \int_0^\infty f_{\Delta | \log w_1}(\delta | x) d\delta / \Pr(\Delta > 0) \\ &= f_{\log w_1}(x) \Pr(\Delta > 0 | \log w_1 = x) / \Pr(\Delta > 0) \end{aligned} \tag{A.9}$$

By the joint normality assumption in (ii), $\Delta | \log w_1 = x$ approximately has the normal distribution with mean $\mu_\Delta + \rho_1 \sigma_\Delta / \sigma$ and variance $(1-\rho_1^2)\sigma_\Delta^2$. Therefore,

$$\Pr(\Delta > 0 | \log w_1 = x) = \Phi\left(\frac{\frac{\mu_\Delta}{\sigma_\Delta} + \rho_1 \frac{x - \mu_1}{\sigma}}{\sqrt{1-\rho_1^2}} \right).$$

Meanwhile, by the log-normality of w_1 and the normality of Δ, we have

$$f_{\log w_1}(x) = \frac{1}{\sigma} \phi\left(\frac{x-\mu_1}{\sigma} \right), \quad \Pr(\Delta > 0) = \Phi\left(\frac{\mu_\Delta}{\sigma_\Delta} \right)$$

The desired result immediately follows. Note that the right hand side of (A.6) is a little bit more general than the density function of skew-normal. It reduces to the skew-normal if $\mu_\Delta = 0$.

If Δ is independent of α, then $f_{\log w_1}(x|\Delta > 0) = f_{\log w_1}(x)$, which has a normal distribution by the result of part (i).

(iii). Observe that

$$\begin{aligned} f_{\log w}(x) &= \Pr(\Delta \geq 0) f_{\log w}(x|\Delta \geq 0) + \Pr(\Delta < 0) f_{\log w}(x|\Delta < 0) \\ &= \Pr(\Delta \geq 0) f_{\log w_1}(x|\Delta \geq 0) + \Pr(\Delta < 0) f_{\log w_0}(x|\Delta < 0), \end{aligned}$$

where $f_{\log w_1}(x|\Delta > 0)$ and $f_{\log w_0}(x|\Delta < 0)$ are given in part (ii).

If $w_1 = w_0$, then $f_{\log w}(x) \approx \frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)$, that is, w approximately has a log-normal distribution.

Q.E.D.

The following lemma is used in the proof of Proposition A1.

LEMMA A1 *Consider random variables X and Y. Assume that (i). $X \sim N(\mu, \sigma)$; (ii). there are small positive numbers t and κ such that* $\Pr(|Y| > t) \leq \kappa$. *Then* $\Pr(X + Y \leq z) \approx \Pr(X \leq z) = \Phi\left(\frac{z-\mu}{\sigma}\right)$.

PROOF: For any $z \in \mathbb{R}$, observe that

$$\begin{aligned} \Pr(X + Y \leq z) &\leq \Pr(X \leq z + t) + \Pr(Y \leq -t) \\ \Pr(X \leq z - t) &\leq \Pr(X + Y \leq z) + \Pr(-Y \leq -t), \end{aligned}$$

Combining the inequalities yields

$$\Pr(X \leq z - t) - \kappa \leq \Pr(X + Y \leq z) \leq \Pr(X \leq z + t) + \kappa.$$

Since X is normal and t, κ is small by assumption, $\Pr(X \leq z - t)$, $\Pr(X \leq z)$ and $\Pr(X \leq z + t)$ are close to one another. Hence, $\Pr(X + Y \leq z) \approx \Pr(X \leq z)$. *Q.E.D.*

3.2. *Log odds ratio of conditional probability*

In the setting of Proposition A1, the log odds ratio

$$\begin{aligned} &\log\left(\frac{\Pr[S(0) = 1|W(0) = w]}{1 - \Pr[S(0) = 1|W(0) = w]}\right) \\ &= \log f_{w_1}(w|\Delta \geq 0) - \log f_{w_0}(w|\Delta < 0) + \log(p/(1-p)) \\ &\approx \log\phi\left(\frac{\log w - \mu_1}{\sigma}\right) - \log\phi\left(\frac{\log w - \mu_0}{\sigma}\right) + \log\Phi\left(\frac{\frac{\mu_\Delta}{\sigma_\Delta} + \rho\frac{\log w - \mu_1}{\sigma}}{\sqrt{1-\rho^2}}\right) \\ &- \log\Phi\left(\frac{-\frac{\mu_\Delta}{\sigma_\Delta} - \rho\frac{\log w - \mu_0}{\sigma}}{\sqrt{1-\rho^2}}\right) + \log(p/(1-p)) \\ &= \log\Phi\left(\frac{\frac{\mu_\Delta}{\sigma_\Delta} + \rho\frac{\log w - \mu_1}{\sigma}}{\sqrt{1-\rho^2}}\right) - \log\Phi\left(\frac{-\frac{\mu_\Delta}{\sigma_\Delta} - \rho\frac{\log w - \mu_0}{\sigma}}{\sqrt{1-\rho^2}}\right) + \frac{\mu_1 - \mu_0}{\sigma^2}\log w + \frac{-\mu_1^2 + \mu_0^2}{2\sigma^2} \\ &+ \log(p/(1-p)). \end{aligned}$$

One can approximate $\Phi(x)$ by Polya's formula $\Phi(x) \approx 0.5\left(1+\sqrt{1-\exp(-2x^2/\pi)}\right)$, which has a maximum error of 0.003 when $x = 1.6$. Overall the log adds ratio can be approximated by a function of $\log w$ with unknown parameters.

3.3. *In the presence of minimum wage*

This section characterizes the distribution of wages in the presence of minimum wage. The results here will be used in Appendix E. In the setting of Proposition A1, $w_0 \approx w_1\xi$ and $r_1 \approx Kw_1 + c$ for some positive constants ξ and K. For a worker previously in the formal sector, i.e., $S(0) = 1$, his/her wage in the presence of the minimum wage can be characterized by

$$W(1) = \begin{cases} w_1, & \text{if } w \geq m, \\ m, & \text{if } \frac{m-c}{K+1} \leq w < m \text{ and } \Delta \geq b(w), \\ w_0, & \text{if } \frac{m-c}{K+1} \leq w < m \text{ and } 0 \leq \Delta < b(w), \text{or } w < \frac{m-c}{K+1} \text{ and } 0 \leq \Delta \leq Kw + c, \\ \cdot & \text{if } w < \frac{m-c}{K+1} \text{ and } \Delta > Kw + c. \text{ (unemployment)} \end{cases}, \tag{A.10}$$

where $b(w) = Kw + c - \sqrt{(Kw+c)^2 - (m-w)^2}$. Let $c^{(1)} \equiv \Pr[S(1) = 1]$, one has

$$c^{(1)} = \int_m^\infty \int_0^\infty f_{w_1,\Delta}(w,\delta)d\delta dw + \int_{\frac{m-c}{K+1}}^m \int_{b(w)}^\infty f_{w_1,\Delta}(w,\delta)d\delta dw.$$

For the workers in the formal sector,

$$\begin{aligned} f_{W(1)}(w|S(1)=1) \quad &= \quad \frac{\mathbb{1}\{w > m\}}{c^{(1)}\sigma w}\phi\left(\frac{\log w - \mu_1}{\sigma}\right)\Phi\left(\frac{\frac{\mu_\Delta}{\sigma_\Delta} + \rho_1\frac{\log w - \mu_1}{\sigma_\Delta}}{\sqrt{1-\rho_1^2}}\right) \\ &\quad + \frac{\mathbb{D}(w-m)}{c^{(1)}\sigma}\int\frac{1}{w}\phi\left(\frac{\log w - \mu_1}{\sigma}\right)\pi_m^{(1)}(w)dw, \end{aligned} \tag{A.11}$$

where

$$\pi_m^{(1)}(w) = \mathbb{1}\{\frac{m-c}{K+1} \leq w < m\}\left[1 - \Phi\left(\frac{\frac{b(w)-\mu_\Delta}{\sigma_\Delta} - \rho_1\frac{\log w - \mu_1}{\sigma_\Delta}}{\sqrt{1-\rho_1^2}}\right)\right].$$

Therefore, the expected wage in the formal sector is

$$\begin{aligned}\mathbb{E}\left[W(1)|S(1)=1\right] &= \int_0^\infty w f_{W(1)}(w|S(1)=1)dw \\ &= \frac{1}{c^{(1)}}\int_m^\infty w\int_0^\infty f_{w_1,\Delta}(w,\delta)d\delta dw + \int_{\frac{m-c}{K+1}}^m \int_{b(w)}^\infty f_{w_1,\Delta}(w,\delta)d\delta dw.\end{aligned}$$

Now consider the informal sector. Let $c^{(0)} \equiv \Pr\left[S(1)=1\right]$.

$$\begin{aligned}c^{(0)} &= \frac{1}{\xi}\int_{\frac{(m-c)\xi}{K+1}}^{m\xi}\int_0^{b(w/\xi)} f_{w_1,\Delta}(w/\xi,\delta)d\delta dw + \frac{1}{\xi}\int_{\max\{0,-c/K\}}^{\frac{(m-c)\xi}{K+1}}\int_0^{Kw/\xi+c} f_{w_1,\Delta}(w/\xi,\delta)d\delta dw \\ &\quad +\frac{1}{\xi}\int_0^\infty\int_{-\infty}^{\min\{0,Kw+c\}} f_{w_1,\Delta}(w/\xi,\delta)d\delta dw.\end{aligned}$$

The observed wage density in the informal sector can be written as

$$f_{W(1)}(w|S(1)=0) = \frac{f_{w_1}(w)}{c^{(0)}\xi}\pi_d^{(1)}(w) + \frac{f_{w_1}(w)}{c^{(0)}\xi}\Phi\left(\frac{\frac{\min\{0,Kw+c\}-\mu_\Delta}{\sigma_\Delta} - \rho_1\frac{\log w-\mu_1}{\sigma_\Delta}}{\sqrt{1-\rho_1^2}}\right),$$

where

$$\begin{aligned}\pi_d^{(1)}(w) &= \mathbb{I}\{\frac{(m-c)\xi}{K+1}\le w < m\xi\}\left[\Phi\left(\frac{\frac{b(w)-\mu_\Delta}{\sigma_\Delta} - \rho_1\frac{\log w-\mu_1}{\sigma_\Delta}}{\sqrt{1-\rho_1^2}}\right) - \Phi\left(\frac{\frac{-\mu_\Delta}{\sigma_\Delta} - \rho_1\frac{\log w-\mu_1}{\sigma_\Delta}}{\sqrt{1-\rho_1^2}}\right)\right] \\ &\quad +\mathbb{I}\{\max\{0,\frac{-c\xi}{K}\} < w < \frac{(m-c)\xi}{K+1}\}\left[\Phi\left(\frac{\frac{Kw+c-\mu_\Delta}{\sigma_\Delta} - \rho_1\frac{\log w-\mu_1}{\sigma_\Delta}}{\sqrt{1-\rho_1^2}}\right) - \Phi\left(\frac{\frac{-\mu_\Delta}{\sigma_\Delta} - \rho_1\frac{\log w-\mu_1}{\sigma_\Delta}}{\sqrt{1-\rho_1^2}}\right)\right.\end{aligned}$$

As a result, the expected wage in the informal sector writes

$$\begin{aligned}\mathbb{E}\left[W(1)|S(1)=0\right] &= \frac{1}{c^{(0)}\xi}\int_{\frac{(m-c)\xi}{K+1}}^{m\xi} w\int_0^{b(w/\xi)} f_{w_1,\Delta}(w/\xi,\delta)d\delta dw \\ &\quad +\frac{1}{c^{(0)}\xi}\int_{\max\{0,-c/K\}}^{\frac{(m-c)\xi}{K+1}} w\int_0^{Kw/\xi+c} f_{w_1,\Delta}(w/\xi,\delta)d\delta dw \\ &\quad +\frac{1}{c^{(0)}\xi}\int_0^\infty w\int_{-\infty}^{\min\{0,Kw+c\}} f_{w_1,\Delta}(w/\xi,\delta)d\delta dw.\end{aligned}$$

The density of the observed wage is

$$f_{W(1)}(w) = \frac{c^{(1)}f_{W(1)}(w|S(1)=1) + c^{(0)}f_{W(1)}(w|S(1)=0)}{c^{(1)}+c^{(0)}},$$

and the expected wage is

$$\mathbb{E}[W(1)] = \frac{c^{(1)}}{c^{(1)}+c^{(0)}}\mathbb{E}\left[W(1)|S(1)=1\right] + \frac{c^{(1)}}{c^{(1)}+c^{(0)}}\mathbb{E}\left[W(1)|S(1)=0\right].$$

4. APPENDIX D: ESTIMATION

4.1. *Likelihood function*

Collecting the results of the previous section, we obtain a tractable expression for the likelihood of the data under the minimum wage policy. Let $\Theta \equiv (\theta, \delta, \pi)$ be the entire vector of model parameters, that is, those governing the latent distribution of wages, the conditional probability of sector given wages and minimum wage effects. Define the likelihood of observing a pair (w, s) given the minimum wage level m and model parameters Θ as $L(W(1) = w, S(1) = s|\Theta) = \Pr[S(1) = s|W(1) = w; \Theta]f(w|\Theta)$.

Three functional forms must be specified to obtain the exact expression for the likelihood function: A functional form for the unconditional distribution of latent wages, one for the conditional probability of latent sector given the latent wage, and, lastly, one functional form for the probability of non-compliance as a function of latent wage. We use lognormality for the latent wage distribution, so $f_0(w) = \frac{1}{w\sigma}\phi(\frac{\log(w)-\mu}{\sigma})$, where ϕ is the density of the standard normal distribution. For the conditional probability of latent sector given the latent wage, $\Pr[S(0) = 1|W(0) = w]$, we use a linear logit specification, so $\Pr[S(0) = 1|W(0) = w] \equiv \Lambda(w) = \frac{e^{\delta_0+\delta_1 w}}{1+e^{\delta_0+\delta_1 w}}$. Finally, for the probability of non-compliance as a function of the wage, we use a constant specification, so $\pi_d^s(w) = \pi_d^{(s)}$ for all w and for $s \in \{0, 1\}$. [4]

Let $\psi(\Theta) \equiv \frac{\pi_m^{(1)} \int^m f_0(w|\theta)\Lambda(w|\delta)dw}{\int^m \pi_m(w) f_0(w|\theta)dw}$. For the first term appearing in the log-likelihood, we have:

$$\begin{aligned}
\log \Pr[S(1) = s|W = w; \Theta] \quad = \quad & \mathbb{1}\{w = m\} \left[\mathbb{1}\{s = 1\} \log \psi(\Theta) + \mathbb{1}\{s = 0\} \log(1 - \psi(\Theta)\right] \\
& + \mathbb{1}\{w > m\} \left[\mathbb{1}\{s = 1\} \log \Lambda(w|\delta) + \mathbb{1}\{s = 0\} \log(1 - \Lambda(w|\delta))\right]. \qquad \text{(A.12)}
\end{aligned}$$

[4]Our main results – namely that the minimum wage generates noticeable disemployment effects, that the size of the formal sector is reduced, and that the likelihood of a formal worker moves to the informal sector as an effect of the minimum wage policy is small, or around 10%– are unchanged when different specifications of these functional forms are used. We tried flexible polynomials for the non-compliance probability, higher-order terms for the logit specification, and more flexible functional forms for the marginal distribution of latent wages. In particular, if anything, we find that the probability of moving to the informal sector is even smaller (and the corresponding probability of unemployment and bunching are larger) when different specifications are used. So we believe that our main qualitative results are robust to the functional forms we used.

For the second term in the log-likelihood, we have:

$$\begin{aligned}\log f(w|\Theta) &= \mathbb{1}\{w<m\}\log(\pi_d(w)f_0(w|\theta))\\ &\quad+\mathbb{1}\{w=m\}\log\Big(\int^m \pi_m(u)f_0(u|\theta)du\Big)\\ &\quad+\mathbb{1}\{w>m\}\log f_0(w|\theta)-\log c(\Theta),\end{aligned} \tag{A.13}$$

where $\pi_d(w) = \Lambda(w|\delta)\pi_d^{(1)} + (1-\Lambda(w|\delta))\pi_d^{(0)}$, and $\pi_m(w) = \Lambda(w|\delta)\pi_m^{(1)} + (1-\Lambda(w|\delta))\pi_m^{(0)}$, and $c(\Theta) \equiv 1-\int^m \pi_u^{(1)}\Lambda(w)f_0(w)dw$. The parameter $c(\Theta)$ can be interpreted as the ratio of employment before and after the introduction of the minimum wage.

Given that $\log L(W(1) = w, S(1) = s|\Theta) = \log \Pr[S(1) = s|W(1) = w;\Theta] + \log f(W(1) = w|\Theta)$, we can define the maximum likelihood estimator of the model parameters as $\widehat{\Theta} = \arg\max_\Theta \frac{1}{N}\sum_i^N \log L(w_i, s_i|\Theta)$.

4.2. *Robustness: Alternative approaches to estimation*

In this section, we discuss alternative approaches to estimate the model that can be useful when one is interested in a greater degree of flexibility in the form that the minimum wage affects certain parts of the wage distribution. Our goals are twofold: To show (i) how one can allow the minimum wage to generate spillover effects higher up on the wage distribution and still obtain consistent estimates of the effects of the policy, and (ii) how one can allow for a much more flexible structure of minimum wage effects in the bottom part of the wage distribution – that is, for workers whose latent wages are below m–. To do that, we will essentially re-write write modified versions of the likelihood function that are consistent with the flexibility we hope to obtain in terms of the model. To start, we will use the following weak characterization of the minimum wage effects, which nests the one we use in the main paper, but allows for much more flexible patterns of effects in the bottom part of the wage distribution:

Remark 3 (Minimum wage effects' structure (weak characterization))

$$Pr[W(1) > km|W(0) < km] = 0, \textit{ for some known } k \geq 1.$$

This assumption does not specify, and, by doing so, it leaves unrestricted the way that the wages of formal sector workers will change when they move to the informal sector, as long as they do not show up too far up in the right tail of the informal sector wage distribution. Here we discuss how one can estimate the model parameters under this weaker condition. It also allows the minimum wage to affect the wages of workers that earn more than the minimum wage, within the range of wages between m and km, where km is a tuning parameter, a user-specified upper bound for the spillover effect.

The maximum likelihood estimator presented in Section 3 takes advantage of the fact that the likelihood function is fully specified under the Dual-Economy extension of Meyer and Wise's model. However, this comes at the cost of restrictive assumptions on how the minimum wage affects the bottom part and the upper part of the wage distribution. That estimator, however, will in general be inconsistent to the true value of the parameters of the model if the minimum wage generates effects higher up on the wage distribution or if the form of minimum wage effects imposed by the effect structure we impose in our main specification (Remarks 1 and 2 in the paper) are incorrect. In this section, we investigate how one can attempt to estimate the model parameters without relying on the complete absence of spillovers (Remark 1) or fully specifying the effects of the minimum wage on the bottom part of the wage distribution (Remark 2). To do that, we rely on the technical condition imposed by Remark 3 above. We will also rely on limited spillovers, that is, we assume that the spillover effects of the minimum wage at a known point km higher up on the wage distribution. Under these assumptions, one can estimate the parameters of the latent wage distribution by means of the following optimization:

$$\widehat{\theta} = \arg\max_{\theta} \frac{1}{N} \sum_{i}^{N} \mathbb{1}\{w_i > km\} \log f(w_i | w_i > km; \theta).$$

Then, estimate the conditional probability of the latent sector given the wages while also using values above km:

$$\widehat{\delta} = \arg\max_{\delta} \frac{1}{N} \sum_{i}^{N} \mathbb{1}\{w_i > km\} \log \Pr[s_i | w_i; \delta].$$

Under Assumption 5, the estimators defined by the equations above will be consistent. This procedure yields consistent estimates because the density of wages for values above the minimum is merely a function of a subset (θ) of the parameter vector (Θ) and the fact that the true parameter θ_0 is the argument that maximizes $\mathbb{E}[\log f(W(1)|W(1) > km; \theta)]$. The same holds for the conditional probability of sector given the wage: The conditional probability of observed sector given the wage, for values above the minimum wage, is only a function of δ and the true parameter δ_0 is the argument that maximizes $\mathbb{E}[\log(\Pr[S(1)|W(1) > km; \delta])]$.

Comparing the latent and observed wage and sector distributions, one can find the effects of the minimum wage on outcomes such as average wages, size of the formal sector, employment, and etc. These estimates obtained using this procedure are robust to spillover effects as long as they are not present above km. These estimates also bypass the need to specify exactly how the minimum wage affects the bottom part of the wage distribution. In particular, we do not need to make any assumption about how the wages of formal sector workers change once they move to the informal sector, or whether the wages of low wage informal sector workers decrease due to the inflow of low wage formal workers to that sector.

5. APPENDIX E: MARGINAL EFFECTS OF THE MINIMUM WAGE

Using the results in Appendix C, Section 3.3, we obtain the following equations that describe the effects of the minimum wage on expected wages,

$$\text{(A.14)} \quad \frac{\partial \mathbb{E}\left[W(1)\right]}{\partial m} = \Pr[W(1) = m] - \frac{1}{c^{(1)} + c^{(0)}} \left(\frac{\partial c^{(1)}}{\partial m} + \frac{\partial c^{(0)}}{\partial m} \right) \mathbb{E}\left[W(1)\right] + \frac{1}{c^{(1)} + c^{(0)}} \left(\frac{\partial c^{(1)}}{\partial m} + \frac{\partial c^{(0)}}{\partial m} \xi \right) m.$$

$$\text{(A.15)} \quad \frac{\partial \mathbb{E}\left[W(1)|S(1) = 1\right]}{\partial m} = \Pr\left[W(1) = m|S(1) = 1\right] - \frac{1}{c^{(1)}} \frac{\partial c^{(1)}}{\partial m} \left\{ \mathbb{E}\left[W(1)|S(1) = 1\right] - m \right\}.$$

$$\text{(A.16)} \quad \frac{\partial \mathbb{E}\left[W(1)|S(1) = 0\right]}{\partial m} = \Pr[W(1) = m|S(1) = 0] - \frac{1}{c^{(0)}} \frac{\partial c^{(0)}}{\partial m} \left\{ \mathbb{E}\left[W(1)|S(1) = 0\right] - \xi m \right\}.$$

$$\text{(A.17)} \quad \frac{\partial c}{\partial m} \approx -\frac{1 - c}{m}.$$

$$\text{(A.18)} \quad \frac{\partial c^{(1)}}{\partial m} \approx -\frac{1 - c^{(1)}}{m}.$$

$$\text{(A.19)} \quad \frac{\partial c^{(0)}}{\partial m} \approx \frac{c^{(0)} - 1}{m}.$$

$$\text{(A.20)} \quad \frac{\partial \log(\Pr[S(1) = 1]/\Pr[S(1) = 0])}{\partial m} = -\frac{1 - c^{(1)}}{c^{(1)}} \frac{1}{m} - \frac{c^{(0)} - 1}{c^{(0)}} \frac{1}{m}.$$

where $c^{(1)} = 1 - \int^{m} (1 - \pi_m^{(1)}) f_0(u|S(0) = 1) du$. The effect of the minimum wage on the average wages of the employed has three key components. The first is the "bite", that is, the proportion of workers who receive the minimum wage. This is the mechanical effect of pushing up the wages of minimum wage workers. The second component concerns unemployment effects: As long as the minimum wage is smaller than the expectation of the observed wages, unemployment will increase the perceived effect of the minimum wage on average wages for those who remain employed. This effect is due to the removal of certain observations at the left tail of the distribution, which contributes to increasing the average wage for those that remain employed. The third effect is related to reallocation of labor across sectors: Some workers will migrate from the formal to the informal sector, and, to the extent that this movement induces changes in their wages, this will contribute to a change in the average.

The model predicts heterogeneous effects of the minimum wage across sectors. The marginal effects of the minimum wage conditional on the sector are a function of the "bite" *and* the coefficients that govern the movements into or out of that sector in response to the policy. The term multiplying $\frac{dc^{(1)}}{dm}$ in (A.15) measures the effect that workers moving out of formality have on expected wages in the sector. The term multiplying $\frac{dc^{(0)}}{dm}$ in (A.16) captures the effect that the entry of workers from the formal sector has on expected wages in the informal sector.[5]

In contrast to the formal sector, the minimum wage has an ambiguous impact on average wages in the informal sector. The minimum wage policy induces an inflow of low-wage workers from the formal sector to the informal sector. This mechanism, depending on the size of the model parameters, can be sufficient to induce an overall reduction of average wages in that sector. In terms of the size of each sector, as long as $\pi_d^{(1)}$ or $\pi_u^{(1)}$ is greater than zero, the minimum wage will reduce the total number of workers employed in the formal sector, and as long as $\pi_d^{(1)}$ is greater than zero, the opposite will be true for the informal sector.

6. APPENDIX F: ADDITIONAL EMPIRICAL RESULTS

6.1. *Robustness*

In this section, we discuss the results we obtain when we estimate the model relaxing some of the assumptions of the model. First, we discuss the results we obtain when we relax the structure imposed by Remark 2 of the paper. There, we specified the effects of the minimum wage on the bottom part of the wage distribution. An important restriction of this assumption is that the wages of workers in the informal sector are not altered by the inflow of formal workers to that sector. Additionally, we also impose that formal workers will earn a wage in the informal sector identical to ξ times the wage that they would earn if they were employed in the formal sector. Under Assumptions 1, 2, and 3, it is possible to estimate the model without imposing these restrictions. To do that, we only need to impose the technical restriction implied by Remark 3 stated above.

[5]Assumptions 3 and 4 exclude general equilibrium effects. The entry of workers into the formal sector is assumed not to change the wages of workers in the informal sector.

The baseline results we present in the main text assume that the minimum wage has no effect on wages above m. This restriction is implied by the structure of the bargaining model. However, this assumption would be violated in several different descriptions of the environment. For example, absence of spillovers will not hold if the technology of production displays complementarity or substitution between workers across different levels of skill. Absence of spillovers will also not hold if workers are matched to firms according to a matching technology that does not differentiate workers by skill level (thus, a change in the number of low wage workers searching for jobs generate congestion externalities for high wage workers, affecting their reservation wages). For all these reasons, it is important to ask whether the results obtained in our preferred specification are substantially changed when we allow for spillover effects.

Table I displays the estimates of the model parameters when we relax the effect structure in the bottom part of the wage distribution (Remark 2). In this specification, the wages of formal workers are allowed to change in a quite flexible manner when they move to the informal sector. Regarding Table I, the qualitative implications of our empirical exercise remain roughly unaltered. The policy generates sizable unemployment effects, the aggregate probability of non-compliance with the policy is approximately 20%, and the latent size of the formal sector is approximately .78.

Table II displays the estimates of the model parameters when we relax the absence of spillovers. In this specification, we allow the minimum wage to have effects on wages up to R$40.00 above the minimum wage.[6] The qualitative results we obtained when we allow for spillover effects are similar to the ones in our preferred specification.

6.2. *Additional Results*

In this section, we report additional results based on our preferred specification and comment on the ability of the model to fit the most important features of the joint

[6]This is roughly two times the average change in the real value minimum wage during the analyzed period. That means, for example, that in the year 2001 we are allowing the minimum wage to have an effect on wages roughly up to the real level of the minimum wage that prevails in the year 2003.

Table I: Robustness - Unrestricted effects on the bottom part of the wage distribution

Parameter		2001	2002	2003	2004	2005
Aggregate						
π_d	Non-compliance	0.19***	0.23***	0.20***	0.21***	0.21***
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
π_m	Bunching	0.26***	0.41***	0.30***	0.27***	0.43***
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
π_u	Non-employment	0.55***	0.36***	0.50***	0.52***	0.36***
		(0.01)	(0.02)	(0.02)	(0.01)	(0.02)
$Pr[W(0) < m]$	Fraction Affected	0.26***	0.24***	0.31***	0.31***	0.29***
		(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
$Pr[S(0) = 1]$	Latent size of the formal sector	0.76***	0.77***	0.78***	0.78***	0.81***
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Formal Sector						
$\pi_d^{(1)}$	Sector mobility	0.03**	0.23***	0.08***	0.08***	0.22***
		(0.01)	(0.02)	(0.02)	(0.01)	(0.02)
$\pi_m^{(1)}$	Bunching	0.21***	0.28***	0.25***	0.23***	0.33***
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$\pi_u^{(1)}$	Non-employment	0.76***	0.48***	0.67***	0.69***	0.46***
		(0.02)	(0.03)	(0.02)	(0.02)	(0.02)
Informal Sector						
$\pi_d^{(0)}$	Non-compliance	0.61***	0.22***	0.54***	0.58***	0.19***
		(0.01)	(0.03)	(0.02)	(0.02)	(0.03)
$\pi_m^{(0)}$	Bunching	0.39***	0.78***	0.46***	0.42***	0.81***
		(0.01)	(0.03)	(0.02)	(0.02)	(0.03)

Note: Bootstrapped standard errors (computed using 100 replications) are given in parentheses.

Table II: Robustness to Spillovers

Parameter		2001	2002	2003	2004	2005
Aggregate						
π_d	Non-compliance	0.18***	0.21***	0.20***	0.20***	0.18***
		(0.00)	(0.01)	(0.01)	(0.01)	(0.02)
π_m	Bunching	0.24***	0.37***	0.28***	0.26***	0.37***
		(0.01)	(0.02)	(0.01)	(0.01)	(0.03)
π_u	Non-employment	0.58***	0.42***	0.52***	0.54***	0.44***
		(0.01)	(0.03)	(0.01)	(0.02)	(0.05)
$Pr[W(0) < m]$	Fraction Affected	0.28***	0.27***	0.33***	0.33***	0.33***
		(0.00)	(0.01)	(0.01)	(0.01)	(0.02)
$Pr[S(0) = 1]$	Latent size of the formal sector	0.78***	0.78***	0.78***	0.79***	0.81***
		(0.00)	(0.00)	(0.00)	(0.00)	(0.01)
Formal Sector						
$\pi_d^{(1)}$	Sector mobility	0.04***	0.20***	0.08***	0.08***	0.18***
		(0.00)	(0.03)	(0.01)	(0.01)	(0.05)
$\pi_m^{(1)}$	Bunching	0.19***	0.24***	0.23***	0.21***	0.27***
		(0.01)	(0.01)	(0.01)	(0.01)	(0.02)
$\pi_u^{(1)}$	Non-employment	0.78***	0.56***	0.69***	0.71***	0.55***
		(0.01)	(0.05)	(0.02)	(0.02)	(0.07)
Informal Sector						
$\pi_d^{(0)}$	Non-compliance	0.62***	0.25***	0.56***	0.59***	0.21**
		(0.01)	(0.07)	(0.01)	(0.02)	(0.10)
$\pi_m^{(0)}$	Bunching	0.38***	0.75***	0.44***	0.41***	0.79***
		(0.01)	(0.07)	(0.01)	(0.02)	(0.10)

Note: Bootstrapped standard errors (computed using 100 replications) are given in parentheses.

Table III: Parameter Estimates

Parameter	2001	2002	2003	2004	2005
Latent wage distribution:					
μ	5.7432***	5.8388***	5.8713***	5.9415***	6.0661***
	(0.0095)	(0.0098)	(0.0089)	(0.0081)	(0.0120)
$\log(\sigma)$	-0.1344***	-0.1459***	-0.1493***	-0.1724***	-0.2038***
	(0.0071)	(0.0076)	(0.0067)	(0.0055)	(0.0077)
Conditional probability of sector given the wage:					
δ_0	0.9066***	1.0692***	1.0441***	1.0554***	1.3348***
	(0.0265)	(0.0229)	(0.0356)	(0.0360)	(0.0382)
δ_1	0.0006***	0.0004***	0.0005***	0.0005***	0.0002***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Note: Bootstrapped standard errors (computed using 100 replications) are given in parentheses.

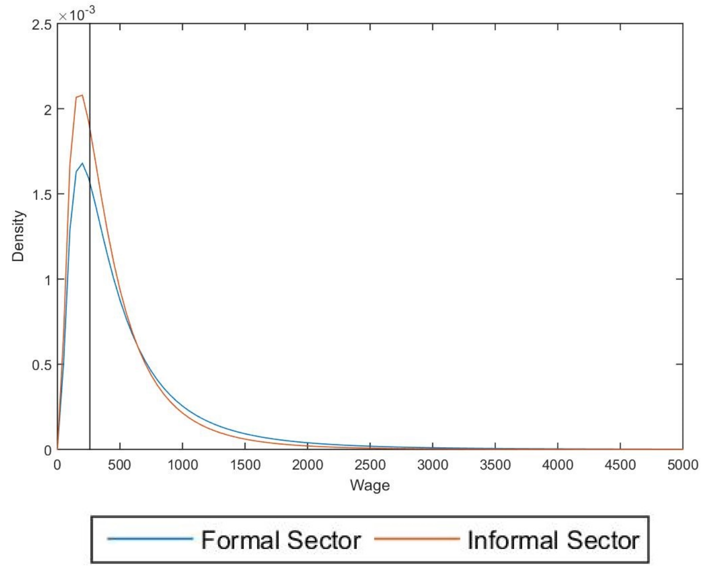
distribution of sector and wage. First, we display the estimated parameters of the latent wage distribution and the conditional probability of formality with respect to the wage. These parameters are not of particular interest on their own but are needed to obtain the marginal effects of the minimum wage and the estimates of the probabilities π.

Figure 1 shows the estimates of the latent wage distributions for the formal and informal sectors. We can see that the informal sector wage distribution tends to have higher density for low wages relative to the formal sector. This follows from the positive estimated slope coefficient on the relationship between latent sectors and wage (δ_1).

Figure 2 shows kernel density estimates of the wage distributions in the formal and informal sectors for the year 2004. We observe substantial differences between the observed wage distributions in the formal and informal sectors. The formal sector wage distribution presents almost no density below the minimum wage level, whereas the informal sector exhibits considerable mass in that range. Above the minimum wage level, the formal sector density tends to be higher than the informal sector density.

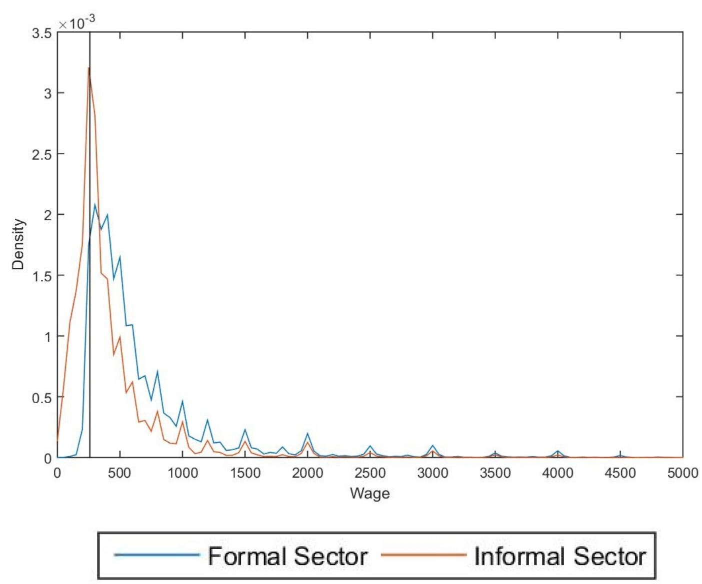
Table IV presents a comparison between certain moments of the data and those implied by the model parameters. In examining this table, we can see that the model can

Figure 1: Latent Densities



Note: Year 2004.

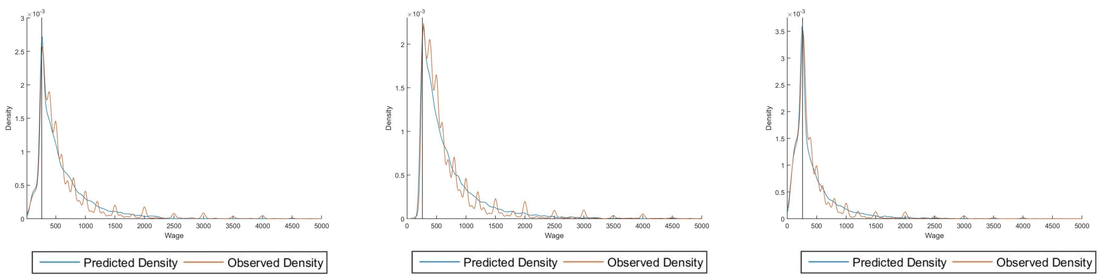
Figure 2: Observed Densities



Note: Density estimates using a gaussian kernel (bandwidth = R$30). Year 2004.

Figure 3: Model Fit

Aggregate Formal Informal

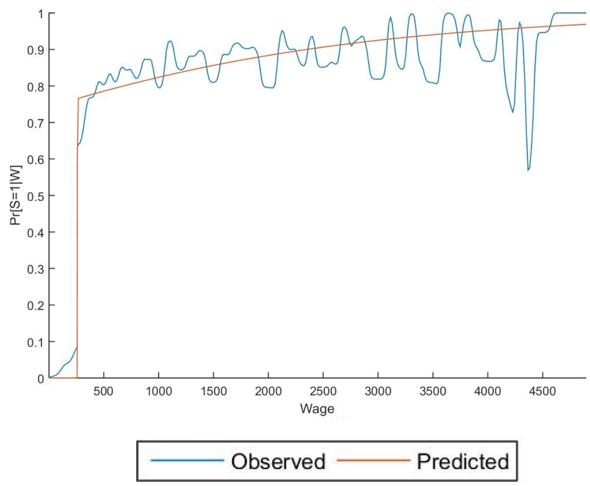


Note: Density estimates based on a gaussian kernel (bandwidth = R$30). Year 2004.

Table IV: Model Fit

Parameter	2001		2002		2003		2004		2005	
	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted
$E[W]$	532.14	520.12	562.16	550.86	606.15	594.30	640.93	628.66	691.47	680.72
$Sd[W]$	538.98	464.06	565.13	487.55	573.33	502.67	587.34	514.75	600.73	531.65
Skewness[W]	3.41	3.19	3.29	3.23	3.12	3.09	3.08	2.98	2.94	2.97
Kurtosis[W]	18.06	17.41	16.70	17.35	15.31	16.05	15.00	15.07	13.91	14.54
$Pr[W = m]$	0.08	0.08	0.10	0.11	0.11	0.11	0.10	0.10	0.14	0.14
$Pr[W < m]$	0.06	0.06	0.06	0.07	0.08	0.08	0.08	0.08	0.07	0.08
$q^{20}[W]$	220.00	244.38	240.00	267.23	254.00	301.61	290.00	321.69	300.00	374.15
$E[W\|S = 1]$	592.12	567.30	628.27	601.66	675.54	643.80	716.31	682.01	760.71	720.95
$E[W\|S = 0]$	380.92	357.95	402.04	360.72	428.16	384.52	446.52	404.88	502.87	425.45
$Sd[W\|S = 1]$	546.56	498.12	572.75	520.83	580.69	534.73	595.35	545.62	601.61	556.42
$Sd[W\|S = 0]$	405.13	335.61	434.38	375.07	429.99	373.40	442.91	380.81	478.16	438.38
$q^{20}[W\|S = 1]$	260.00	264.06	280.00	297.83	300.00	341.77	350.00	361.85	380.00	414.51
$q^{20}[W\|S = 0]$	180.00	175.50	196.00	185.62	200.00	191.16	200.00	201.20	240.00	243.00
$Pr[S = 1]$	0.72	0.71	0.71	0.70	0.72	0.71	0.72	0.71	0.73	0.72
$Pr[S = 1\|W < m]$	0.08	0.00	0.09	0.00	0.09	0.00	0.05	0.00	0.04	0.00
$Pr[S = 1\|W = m]$	0.58	0.57	0.51	0.48	0.62	0.60	0.62	0.60	0.60	0.56
$Pr[S = 1\|W > m]$	0.77	0.77	0.78	0.78	0.79	0.79	0.80	0.80	0.82	0.82

Figure 4: Conditional Probability of Sector Given the Wage



Note: Observed conditional probabilities based on a non-parametric local constant (Nadaraya-Watson) estimator using a gaussian kernel (bandwidth = R\$30).Year 2004.

capture most of the features of the joint distribution of sector and wage. It predicts the discontinuous shape of $\Pr[S(1) = 1|W(1)]$ observed in the data. It fits the probabilities of observing wages at and below the minimum wage level and explains the differences observed between the formal and informal sector distributions. Interestingly, the model can match higher moments of the wage distribution, such as skewness and kurtosis. This need not be the case in general, especially if the parametric family for the wage distribution is severely misspecified.

Figure 4 shows the observed and predicted conditional probabilities of formality given the wage. Figure 3 displays the predicted and observed densities of the aggregate wage distribution and the formal and informal sector distributions. By examining these figures, we again see that the model matches most of the prominent features of the data, except the "heaping" observed at round numbers. It is nevertheless interesting to note the resemblance between the predicted and observed curves in the empirical cumulative distribution for the formal and informal sectors, particularly at and below the minimum wage level.

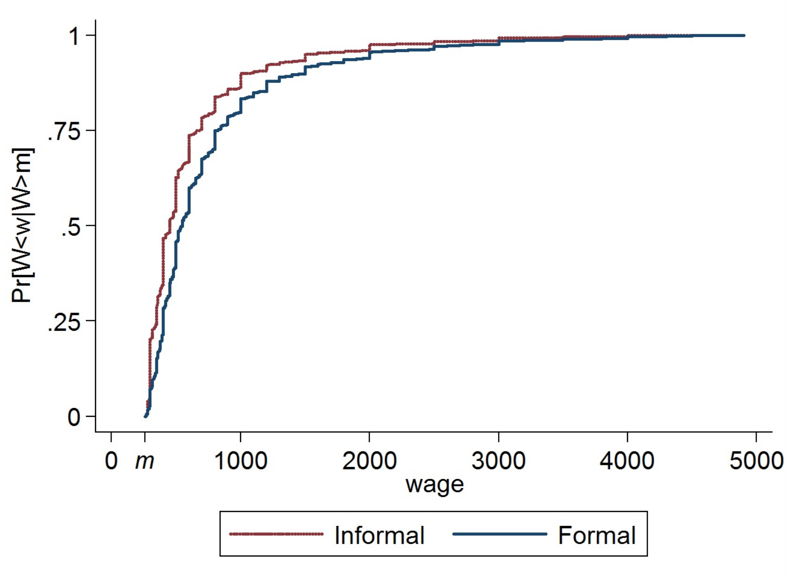
6.2.1. *Decomposing the Differences in the Wage Distributions Across Sectors – The Role of the Minimum Wage*

Figure 5 displays the empirical CDF for the formal and informal sector for wages above the minimum wage. These estimates are, by construction, invariant to the minimum wage effects on the formal and informal sectors for values below the minimum wage. Note that we observe a substantially smaller difference in the CDFs across sectors in Figure 5 than in Figure 6 in the main article. This exercise suggests that the differences across sectors observed in the upper part of the wage distribution across sectors are also a consequence of the effects of the minimum wage in the bottom part of the wage distribution.

The estimates of the model parameters allow us to understand the differences between the wage distributions in the formal and informal sectors. Let $D_1 \equiv f(w|S(1) = 1) - f(w|S(1) = 0)$. That is, D_1 is defined as the observed difference in the density of wages between the formal and informal sectors. Let $D_m \equiv [f(w|S(1) = 1) - f_0(w|S(0) = 1)] - [f(w|S(1) = 0) - f_0(w|S(0) = 0)]$. That is, D_m is defined as the difference in the effects of the minimum wage between the formal and informal sectors. Define $D_0 \equiv f_0(w|S(0) = 1) - f_0(w|S(0) = 0)$. That is, D_0 is defined as the difference between the *latent* wage densities between formal and informal sectors. From the definitions, we have $D_1 = D_m + D_0$. Given the estimates of the model parameters, it is possible to compute D_1, D_m and D_0. By comparing these estimates, we can infer the extent to which the differences in the wage densities between the formal and informal sectors is due to the minimum wage versus differences that would be present regardless of the minimum wage policy. This decomposition can be performed at every point of the wage distribution.

Figure 6 displays the differences in the density of wages between the formal and informal sectors. Given that the observed wage distribution in the formal sector stochastically dominates that in the informal sector, we observe a negative difference between the formal and informal wage densities for low wages and positive for high wages.

Figure 5: Empirical CDFs Above the Minimum Wage by Sector

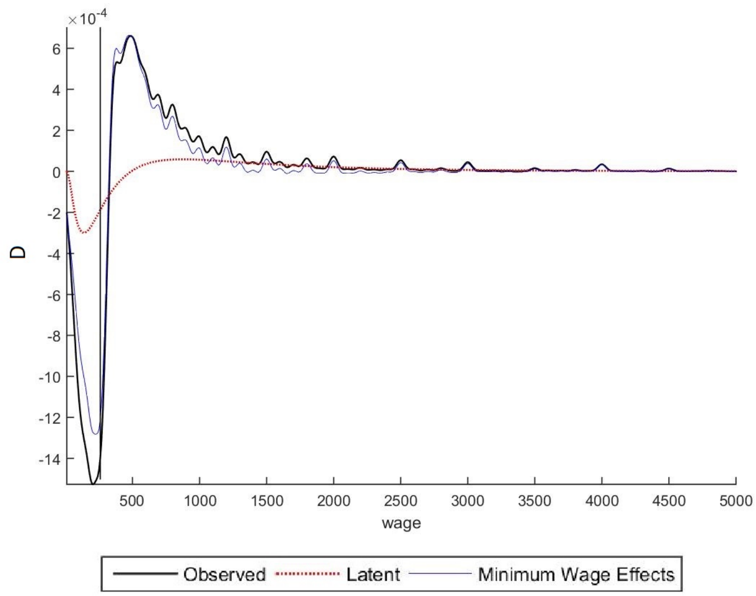


We observe a similar pattern for the latent density as well. Figure 6 also displays the differences between the effects of the minimum wage at each point of the wage distribution. The estimates of the differences in the minimum wage effect tend to closely follow the differences in the observed wage density across sectors. If we decompose the differences in the observed wage distributions between the formal and informal sectors in differences in latent wage distributions and differences in minimum wage effects, my estimates suggest a larger role for the latter. For example, at the 5th quantile of the wage distribution, the minimum wage policy accounts for 82% of the differences in the wage density across sectors.

A decomposition of the difference in quantiles across sectors can be performed in a similar way. The minimum wage accounts for 93% of the differences in the 10th percentile of the wage distribution across sectors. Our estimates imply that 72% of the differences in the median of the wage distribution between the formal and the informal sectors is due to the minimum wage. At higher percentiles, such as the 90th percentile, the minimum wage still accounts for 68% of the differences observed between the formal and informal sectors. The minimum wage accounts for a substantial part of

Figure 6: Decomposition of the Differences of the Densities Across Sectors

Formal minus Informal



Note: Density estimates based on a gaussian kernel (bandwidth = R\$30). Year 2004.

the differences in the wage distribution across sectors for low *and* high wages, even when wages above the minimum wage are, by assumption, not affected by the policy. The reason for a substantial role of the minimum wage on explaining the differences in the formal and informal sector wage densities above the minimum wage is the opposite ways that the wage densities are rescaled across sectors due to the inflow/outflow of workers as a result of the minimum wage policy.